Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2021

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. The points P and Q lie 1 km apart on a straight level road. A car passes P with a speed of 2 m s^{-1} and accelerates with a uniform acceleration of 2.5 m s^{-2} for 8 seconds to a speed of v m s^{-1}. It then travels at this constant speed of v m s^{-1} for 18 seconds. Finally, the car decelerates uniformly to rest at Q.
Calculate

(i) the speed v
(ii) the distance travelled in the first 8 seconds
(iii) the distance travelled in the next 18 seconds
(iv) the total time it takes the car to travel from P to Q.

Later, a motorbike passes P with a speed of k m s^{-1} and continues at this speed for 10 seconds. It then accelerates uniformly for a further 2 seconds to a speed of 17 m s^{-1}. It continues at this speed until it passes Q. The motorbike takes one minute to travel from P to Q.

(v) Draw a speed-time graph of the motion of the motorbike from P to Q.
(vi) Find the value of k.

(i)

$$v = u + at$$
$$= 2 + 2.5 \times 8$$
$$v = 22 \text{ m s}^{-1}$$
(10)

(ii)

$$s = ut + \frac{1}{2}at^2$$
$$s = 2(8) + \frac{1}{2}(2.5)\left(8^2\right)$$
$$s = 96 \text{ m.}$$
(10)

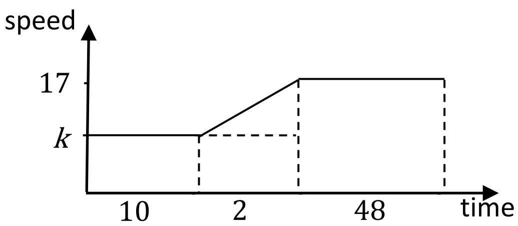
(iii)

$$s = ut + \frac{1}{2}at^2$$
$$= 22 \times 18 + 0$$
$$= 396 \text{ m.}$$
(10)

(iv)

$$96 + 396 + \frac{1}{2}t(22) = 1000$$
$$t = 46.2$$
$$\Rightarrow \text{time} = 8 + 18 + 46.2 = 72.2 \text{ s.}$$
(10)

(v)



(5)

(vi)

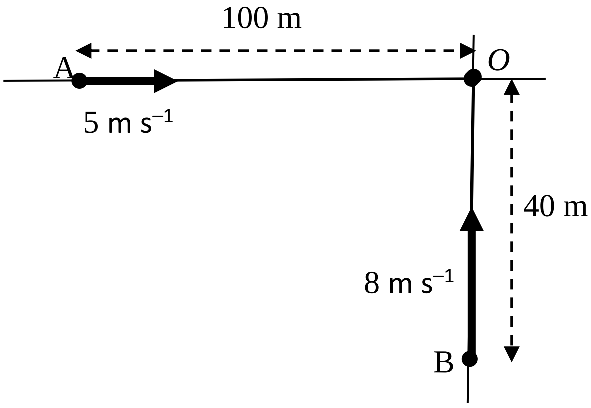
$$12k + \frac{1}{2}(2)(17 - k) + 48 \times 17 = 1000$$
$$11k + 833 = 1000$$
$$k = 15.2$$
(5) (50)

2. **(a)** Two cars, A and B, are located 100 m and 40 m respectively from junction *O*, as shown in the diagram. Car A is travelling east at 5 m s^{-1} and car B is travelling north at 8 m s^{-1}.

(i) Calculate the time it takes B to reach the junction *O*.

(ii) Write the velocity of car A and car B in terms of $\vec{i}$ and $\vec{j}$.

(iii) Find the velocity of A relative to B in magnitude and direction.



(i) $t = \frac{40}{8} = 5 \text{ s}$ (5)

(ii) $\vec{V_A} = 5\,\vec{i} + 0\,\vec{j}$ (5)

$\vec{V_B} = 0\,\vec{i} + 8\,\vec{j}$ (5)

(iii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$\vec{V}_{AB} = 5\,\vec{i} - 8\,\vec{j}$ (5)

$|\vec{V}_{AB}| = \sqrt{5^2 + (-8)^2} = \sqrt{89}$ (5)

$\tan\alpha = \frac{8}{5} \Rightarrow \alpha = 58°$ (5)

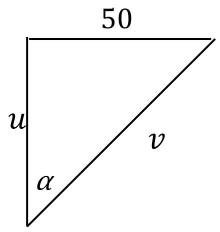
(b) An airport, *D*, is located 400 km north of a second airport *C*. In still air, an aircraft can fly from *C* to *D* in 1 hour 42 minutes. On a particular day the wind is blowing from the east with a velocity of 50 km h^{-1}. The aircraft leaves airport *C* at 13:00.

Calculate

(i) the direction in which the aircraft heads so that it lands at airport *D*

(ii) the time that the aircraft will land at airport *D*.

(i)



$v = \frac{400}{1.7} = \frac{4000}{17} = 235.29$ (5)

$\sin\alpha = \frac{50}{235.29} \Rightarrow \alpha = 12.27°$ (5)

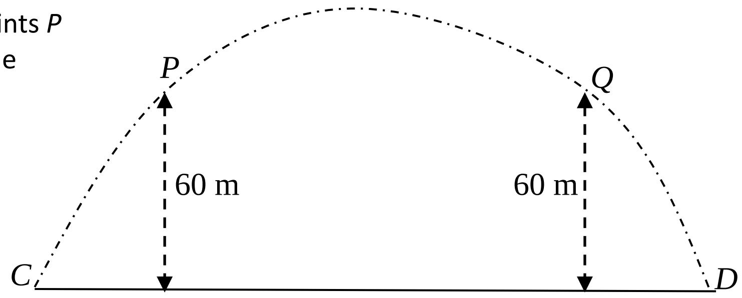
(ii) $u = \sqrt{235.29^2 - 50^2} = 229.92$ (5)

$t = \frac{400}{229.92} = 1.74 \text{ h}$

$\Rightarrow$ time 14:44. (5) (50)

3. **(a)** A particle is projected with an initial velocity of $26\vec{\imath} + 40\vec{\jmath}$ m s^{-1} from a point C on a horizontal plane. The particle lands at point D.

The particle passes through points P and Q which are 60 m above the horizontal plane.



Calculate

(i) the time of flight

(ii) the horizontal range

(iii) the times when the particle is 60 m above the horizontal plane

(iv) the distance between points P and Q.

(b) A particle is projected from the top of a vertical cliff with an initial velocity of 35 m s^{-1} at an angle of α above the horizontal, where $\tan\alpha = \frac{3}{4}$. The cliff is 98 m high and the particle lands a distance of x m from the foot of the cliff.

Calculate

(i) the time it takes for the particle to land

(ii) the value of x.

(a) **(i)** $\vec{r}_j = 0 \quad \Rightarrow \quad 40t - 5t^2 = 0$

$t = 8$ s (10)

(ii) $\text{Range} = 26 \times 8 = 208$ m (10)

(iii) $\vec{r}_j = 60 \quad \Rightarrow \quad 40t - 5t^2 = 60$

$t^2 - 8t + 12 = 0$

$t = 2, \quad t = 6$ (10)

(iv) $|PQ| = 26 \times (6 - 2) = 104$ m (5)

(b) **(i)** $\vec{r}_j = -98$

$-98 = 35\sin\alpha \times t - 5t^2$

$5t^2 - 21t - 98 = 0$

$t = 7$ (10)

(ii) $x = 35\cos\alpha \times t$

$x = 35 \times \left(\frac{4}{5}\right) \times 7 = 196$ m (5) (50)

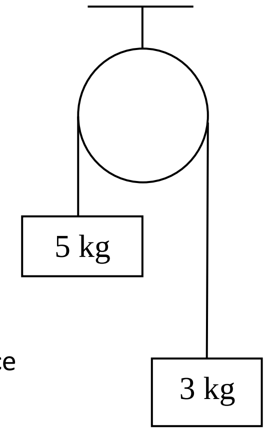
4. **(a)** Masses of 5 kg and 3 kg are connected by a light inelastic string which passes over a smooth light pulley.
The system is released from rest.

(i) Show the forces acting on each mass.

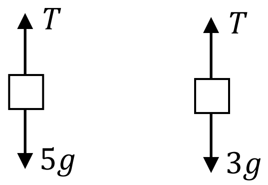
(ii) Find the common acceleration of the system.

(iii) Find the tension in the string.

(iv) Calculate the speed of the 5 kg mass after it has travelled a distance of 2 m.



(i)



(10)

(ii)

$$5g - T = 5a$$

$$T - 3g = 3a$$

$$a = 2.5\ \text{m s}^{-2}$$ (10)

(iii)

$$T = 3a + 3g$$

$$T = 37.5\ \text{N}$$ (5)

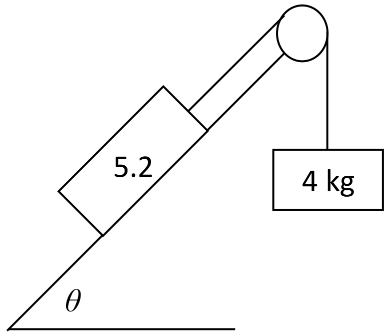
(iv)

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 2.5 \times 2$$

$$v = \sqrt{10} = 3.16\ \text{m s}^{-1}$$ (5) (30)

(b) Masses of 5.2 kg and 4 kg are connected by a light inelastic string which passes over a light smooth pulley, as shown in the diagram. The 5.2 kg mass lies on a rough plane inclined at θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The coefficient of friction μ is $\frac{1}{4}$.

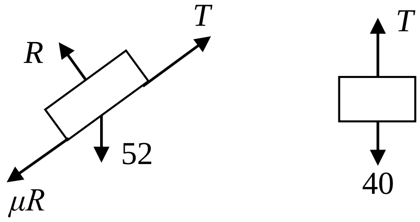


The 4 kg mass hangs vertically and accelerates downwards when the system is released from rest.

(i) Show on separate diagrams the forces acting on each mass.

(ii) Find the common acceleration of the system.

(i)



(10)

(ii)

$$40 - T = 4a \qquad (5)$$

$$T - 52\sin\theta - \mu R = 5.2a$$

$$T - 52 \times \frac{5}{13} - \frac{1}{4} \times 52 \times \frac{12}{13} = 5.2a$$

$$40 - 32 = 9.2a$$

$$a = \frac{20}{23} = 0.87 \text{ m s}^{-2}. \qquad (5) \qquad (20)$$

5. **(a)** Smooth spheres P and Q are travelling towards each other on a smooth horizontal table. P has a mass of 7 kg and Q has a mass of 3 kg. Their speeds before collision are 1 m s^{-1} and 5 m s^{-1} respectively, as shown in the diagram.

The coefficient of restitution for the collision is $\frac{2}{3}$.

Find **(i)** the speeds of P and Q immediately after the collision

(ii) the loss of kinetic energy due to the collision

(iii) the magnitude of the impulse imparted to P as a result of the collision.

(b) A ball is dropped from a height of 11.25 m. The ball strikes a smooth horizontal floor and rises to a vertical height of *h* m. The coefficient of restitution between the ball and the floor is $\frac{3}{5}$.

Calculate

(i) the velocity of the ball immediately after it hits the floor

(ii) the value of *h*.

(a) **(i)** PCM $7(1) + 3(-5) = 7v_1 + 3v_2$ (5)

$$7v_1 + 3v_2 = -8$$

NEL $v_1 - v_2 = -\frac{2}{3}(1+5)$ (5)

$$3v_1 - 3v_2 = -12$$

$v_1 = -2 \qquad v_2 = 2$ (10)

(ii) $\text{KE}_\text{B} = \frac{1}{2}(7)(1)^2 + \frac{1}{2}(3)(5)^2 = 41$

$\text{KE}_\text{A} = \frac{1}{2}(7)(-2)^2 + \frac{1}{2}(3)(2)^2 = 20$

$\text{KE}_\text{B} - \text{KE}_\text{A} = 41 - 20 = 21 \text{ J}$ (5)

(iii) $I = |7 \times (-2) - 7 \times (1)| = 21 \text{ N m}$ (5)

(b) **(i)** $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 10 \times 11.25$

$v = 15$ (10)

$v_r = ev = \frac{3}{5} \times 15 = 9 \text{ m s}^{-1}$ (5)

(ii) $v^2 = u^2 + 2as$

$0 = 81 - 20h$

$h = 4.05 \text{ m}$ (5) (50)

6. **(a)** Particles of weight 3 N, 1 N, 4 N, and 2 N are placed at the points (p, q), (1, p), (q, 4), and (0, 3) respectively.

The co-ordinates of the centre of gravity of the system are (–0.5, 1.5).

Find **(i)** the value of p

(ii) the value of q.

(i) $-\frac{1}{2} = \frac{3p+1+4q+0}{10}$ (5)

$3p + 4q = -6$

$\frac{3}{2} = \frac{3q+p+16+6}{10}$ (5)

$p + 3q = -7$

$p = 2$ (5)

(ii) $3p + 4q = -6$

$6 + 4q = -6$

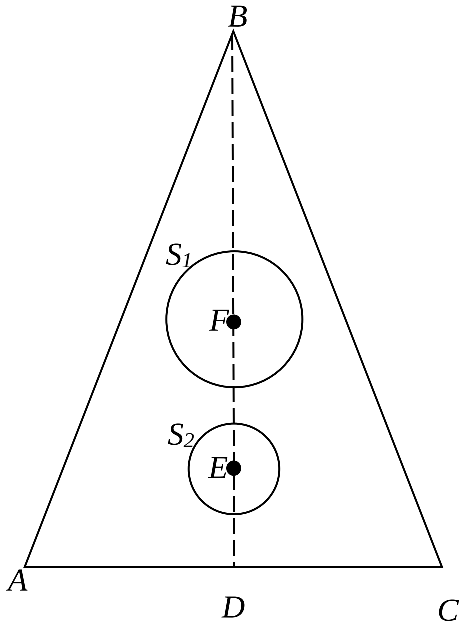
$q = -3$ (5) (20)

(b) A uniform triangular lamina in the shape of an isosceles triangle has the vertices *A*, *B* and *C*. $|AB| = |BC| = 25$ cm and $|AC| = 14$ cm. $|BD|$ is the perpendicular height of the triangle, where *D* is the midpoint of *AC*.

Two uniform circles, S_1 and S_2, are removed from the triangular lamina. S_1 has a radius of 3 cm and its centre *F,* is on the midpoint of *BD*. S_2 has a radius of 2 cm and centre *E* on *BD*, where $|DE| = 6$ cm.

(i) Taking the coordinates of the vertex *A* to be (0, 0), write down the coordinates of the points *B*, *C*, *E* and *F*.

(ii) Calculate the coordinates of the centre of gravity of the remaining shape.



(i) $|BD| = \sqrt{25^2 - 7^2} = 24$

B (7, 24) C (14, 0)

E (7, 6) F (7, 12) (10)

		area	c.g.	
(ii)	*ABC*	$\frac{1}{2}(14)(24) = 168$	$(7,\ 8)$	(5)
	S_1	9π	$(7,\ 12)$	
	S_2	4π	$(7,\ 6)$	(5)
	ABCDE	$168 - 13\pi$	$(7,\ y)$	(5)

$$(168 - 13\pi)y = 168 \times 8 - 9\pi \times 12 - 4\pi \times 6$$

$$(168 - 13\pi)y = 1344 - 132\pi$$

$$y = 7.31$$

$$(7, 7.31)$$ (5) (30)

7. **(a)** A uniform beam *AB* of length 5 m and weight 1000 N is held in a horizontal position by two vertical forces, F_1 and F_2, positioned at *A* and *C* respectively. *C* is 1 m from *B*. The beam is in equilibrium and stationary.

Find **(i)** the value of F_1

(ii) the value of F_2.

(i) C: $F_1 \times 4 = 1000 \times 1.5$

$$F_1 = 375 \text{ N} \qquad (10)$$

(ii)

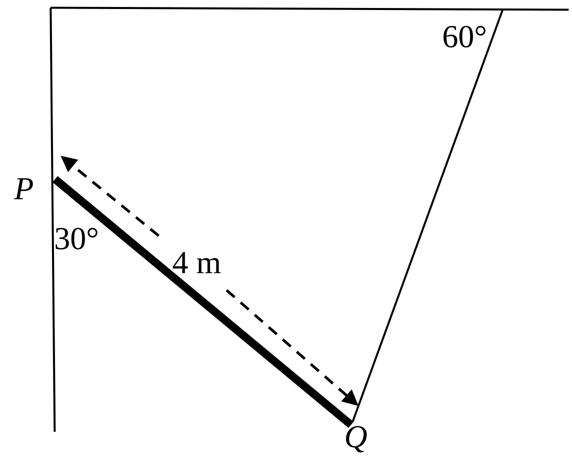
$$F_1 + F_2 = 1000$$

$$375 + F_2 = 1000$$

$$F_2 = 625 \text{ N} \qquad (10) \qquad (20)$$

(b) A uniform rod PQ of length 4 meters and weight 120 N is smoothly hinged at end P to a vertical wall. One end of a light inelastic string connects to Q and the other end of the string is connected to a horizontal ceiling. The rod makes an angle of 30° with the vertical wall and the string makes an angle of 60° with the ceiling.

The rod is in equilibrium.



(i) Show on a diagram all the forces acting on the rod.

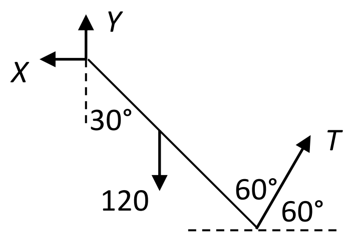
(ii) Write down the equations that arise from resolving the forces horizontally and vertically.

(iii) Write down the equation that arises from taking moments about the point P.

(iv) Calculate the tension in the string.

(v) Calculate the horizontal and vertical components of the reaction at the hinge.

(i)



(5)

(ii) $X = T\cos 60$ (5)

$Y + T\sin 60 = 120$ (5)

(iii) $T\sin 60 \times 4 = 120 \times 2\sin 30$ (5)

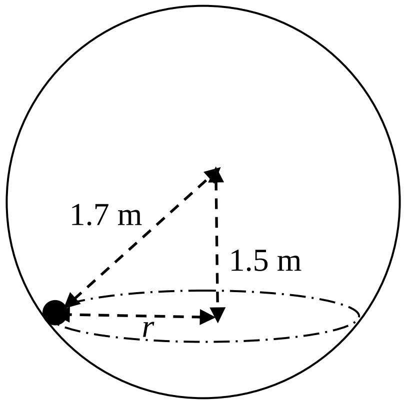
(iv) $T\sin 60 \times 4 = 120 \times 2\sin 30$

$T \times 2\sqrt{3} = 120$

$T = 20\sqrt{3}\ \text{N}$ (5)

(v) $X = 10\sqrt{3} \qquad Y = 90$ (5) (30)

8. **(a)** A smooth particle of mass 2 kg performs uniform horizontal circular motion on the inside surface of a smooth hollow sphere of radius 1.7 m. The radius of the circular motion is *r* m. The centre of the circle is 1.5 m below the centre of the sphere.



(i) Calculate the value of *r*.

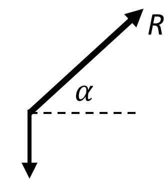
(ii) Draw a diagram showing all the forces acting on the particle.

(iii) Calculate the reaction between the particle and the sphere.

(iv) Calculate the speed of the particle.

(i) $r = \sqrt{1.7^2 - 1.5^2} = 0.8$ m (5)

(ii)



(5)

(iii) $R\sin\alpha = 20$ (5)

$R \times \frac{1.5}{1.7} = 20$

$R = 22.7$ N (5)

(iv) $R\cos\alpha = \frac{mv^2}{r}$ (5)

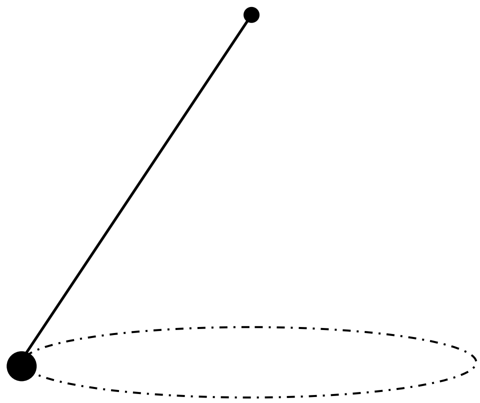
$\frac{68}{3} \times \frac{8}{17} = \frac{2 \times v^2}{0.8}$

$v = 2.07 \ \ \mathrm{m\,s^{-1}}$ (5) (30)

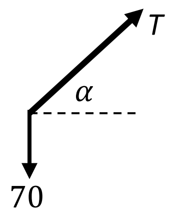
(b) A 7 kg object is attached to a fixed point by a light inelastic string of length 5 m. The object moves in a uniform horizontal circle of radius 3 m.

(i) Draw a force diagram showing all the forces acting on the object.

(ii) Find the angular velocity of the object.



(i)



(5)

(ii)

$T \sin\alpha = 70$ (5)

$T \times \frac{4}{5} = 70$

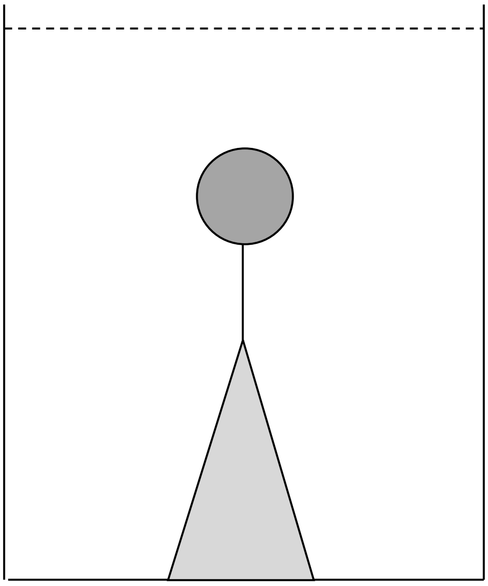
$T = 87.5\ \text{N}$ (5)

$T \cos\alpha = \frac{mv^2}{r}$

$87.5 \times \frac{3}{5} = \frac{7 \times v^2}{3}$

$v = 22.5\ \ \text{m s}^{-1}$ (5) (20)

9. A right circular cone of radius 12 cm and height 25 cm is placed at the bottom of a tank of water. A sphere, of radius 4 cm is attached to the top of the cone with a light inelastic string. The relative density of the cone is 1.8 and the relative density of the sphere is 0.7.
Both the cone and the sphere are fully submerged in water and are at rest.

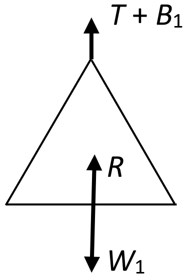


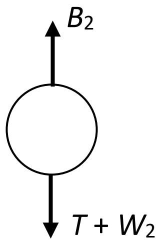
(i) Show on separate diagrams the forces acting on the cone and on the sphere.
(ii) Write equations to represent all the forces acting on the cone and the sphere.
(iii) Calculate the tension in the string.
(iv) Calculate the value of the reaction force between the base of the cone and the bottom of the tank.

The pressure at the top of the sphere is 4000 Pa less than the pressure at the bottom of the tank.

(v) Calculate the length of the string.

(i)





(10)

(ii)

$$R + T + B_1 = W_1 \qquad (5)$$

$$B_2 = T + W_2 \qquad (5)$$

(iii)

$$B_2 = T + W_2$$

$$1000Vg = T + 700Vg \qquad (5)$$

$$T = 300 \times \left(\frac{4}{3} \times \pi \times 0.04^3\right) \times 10$$

$$T = \frac{704}{875} = 0.80 \text{ N} \qquad (5)$$

(iv)

$$R + T + B_1 = W_1$$

$$R + 0.80 + 1000V_1 g = 1800V_1 g \qquad (5)$$

$$R + 0.80 = 800 \times \left(\frac{1}{3} \times \pi \times 0.12^2 \times 0.25\right) \times 10$$

$$R = 29.36 \text{ N} \qquad (5)$$

(v)

$$P_1 = 4000 + P_2$$

$$1000 \times 10 \times h_1 = 4000 + 1000 \times 10 \times h_2 \qquad (5)$$

$$h_1 - h_2 = 0.4$$

$$0.25 + L + 0.08 = 0.4$$

$$L = 0.07 \text{ m} \qquad (5) \qquad (50)$$