10 (b) The acceleration of a cyclist freewheeling down a slight hill is

2010

$$0.12 - 0.0006v^2 \ \text{m s}^{-2}$$

where the velocity v is in metres per second.

The cyclist starts from rest at the top of the hill.

Find **(i)** the speed of the cyclist after travelling 120 m down the hill

(ii) the time taken by the cyclist to travel the 120 m if his average speed is 2.65 m s^{-1}.

(i)

$$v\frac{dv}{dx} = 0.12 - 0.0006v^2$$

$$\int_0^v \frac{v}{0.12 - 0.0006v^2}\,dv = \int_0^{120} dx \qquad 5$$

$$\left[-\frac{1}{0.0012}\ln\left(0.12 - 0.0006v^2\right)\right]_0^v = [x]_0^{120} \qquad 5$$

$$-\frac{1}{0.0012}\ln\left(0.12 - 0.0006v^2\right) + \frac{1}{0.0012}\ln(0.12) = 120 \qquad 5$$

$$\frac{1}{0.0012}\ln\left(\frac{0.12}{0.12 - 0.0006v^2}\right) = 120$$

$$\ln\left(\frac{0.12}{0.12 - 0.0006v^2}\right) = 0.144 \qquad 5$$

$$\frac{0.12}{0.12 - 0.0006v^2} = e^{0.144} = 1.155$$

$$\Rightarrow v = 5.18 \ \text{m s}^{-1} \qquad 5$$

(ii)

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

$$2.65 = \frac{120}{t}$$

$$\Rightarrow t = 45.3 \ \text{s} \qquad 5$$

30