3 (b) A plane is inclined at an angle $60°$ to the horizontal. A particle is projected up the plane with initial speed u at an angle θ to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.

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The particle strikes the plane at right angles.

Show that the range on the inclined plane is $\dfrac{4\sqrt{3}\,u^2}{13g}$.

$r_j = 0$

$$0 = u\sin\theta . t - \tfrac{1}{2} g\cos 60 . t^2$$

$$\Rightarrow \quad t = \frac{2u\sin\theta}{g\cos 60} \quad \text{or} \quad \frac{4u\sin\theta}{g}$$ 5 5

$v_i = 0$

$$0 = u\cos\theta - g\sin 60 . t$$

$$\Rightarrow \quad t = \frac{u\cos\theta}{g\sin 60} \quad \text{or} \quad \frac{2u\cos\theta}{g\sqrt{3}}$$ 5

$$t = \frac{4u\sin\theta}{g} = \frac{2u\cos\theta}{g\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{1}{2\sqrt{3}}, \quad \sin\theta = \frac{1}{\sqrt{13}} \quad \text{and} \quad \cos\theta = \frac{2\sqrt{3}}{\sqrt{13}}$$ 5

$$\text{Range} = u\cos\theta\left\{\frac{4u\sin\theta}{g}\right\} - \frac{1}{2} g\sin 60 . \left\{\frac{4u\sin\theta}{g}\right\}^2$$ 5

$$= \frac{8u^2\sqrt{3}}{13g} - \frac{4u^2\sqrt{3}}{13g}$$

$$= \frac{4u^2\sqrt{3}}{13g}$$ 5 30