10 **(b)** A particle of mass m is projected vertically upwards with speed u. The air resistance is kv^2 per unit mass when the speed is v.

The maximum height reached by the particle is $\dfrac{\ln 4}{2k}$.

(i) Find the value of u in terms of k.

(ii) Find the value of k if the time to reach the greatest height is $\dfrac{\pi}{3}$ seconds.

(i)

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$-mg - mkv^2 = m\frac{vdv}{dx} \qquad 5$$

$$v\frac{dv}{dx} = -\left(g + kv^2\right)$$

$$-\int_u^0 \frac{v}{g + kv^2}\,dv = \int_0^{\frac{\ln 4}{2k}} dx \qquad 5$$

$$\left[-\frac{1}{2k}\ln\left(g + kv^2\right)\right]_u^0 = \frac{\ln 4}{2k} \qquad 5$$

$$-\frac{1}{2k}\ln(g) + \frac{1}{2k}\ln\left(g + ku^2\right) = \frac{\ln 4}{2k}$$

$$\ln\left(\frac{g + ku^2}{g}\right) = \ln 4$$

$$\Rightarrow u = \sqrt{\frac{3g}{k}} \qquad 5$$

(ii)

$$\frac{dv}{dt} = -\left(g + kv^2\right)$$

$$-\int_u^0 \frac{1}{g + kv^2}\,dv = \int_0^{\frac{\pi}{3}} dt$$

$$\left[-\frac{1}{k}\frac{1}{\sqrt{\frac{g}{k}}}\tan^{-1}\left(\frac{v}{\sqrt{\frac{g}{k}}}\right)\right]_{\sqrt{\frac{3g}{k}}}^0 = \frac{\pi}{3}$$

$$\frac{1}{\sqrt{gk}}\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

$$\Rightarrow k = \frac{1}{g} \qquad 5 \qquad 25$$