

2008 10. (a) If

$$x^2 y \frac{dy}{dx} + y \frac{dy}{dx} = 1$$

and $y = 0$ when $x = 0$, find the value of x when $y = \sqrt{\frac{\pi}{2}}$.

$$x^2 y \frac{dy}{dx} + y \frac{dy}{dx} = 1$$

$$y \frac{dy}{dx} (1 + x^2) = 1$$

$$\int y dy = \int \frac{1}{1+x^2} dx$$

$$\frac{1}{2} y^2 = \tan^{-1} x + C$$

$$y = 0, x = 0 \Rightarrow C = 0$$

$$\frac{1}{2} y^2 = \tan^{-1} x$$

$$y = \sqrt{\frac{\pi}{2}}$$

$$\Rightarrow \frac{\pi}{4} = \tan^{-1} x$$

$$\Rightarrow x = 1$$

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