(b) A uniform disc of mass m and radius r rolls from rest, without sliding, 30 m down a plane inclined at an angle of 30° to the horizontal.

(i) Find the linear speed of the disc after rolling 30 m down the plane.

(ii) Find the time taken for the disc to roll 30 m down the plane, correct to two places of decimals.

(iii) The disc is now replaced by a hoop of mass m and radius r. The hoop rolls from rest, without sliding, 30 m down the plane. Show that the ratio of the acceleration down the plane of the hoop to that of the disc is $\frac{3}{4}$.

(i)

$$\text{Gain in KE} = \text{Loss in PE}$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left\{\tfrac{1}{2}mr^2\right\}\omega^2 = mg\{30\sin 30\}$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{4}mv^2 = 15mg$$

$$v^2 = 20g$$

$$v = \sqrt{20g} \text{ or } 14\ \text{ms}^{-1}$$

5

5

(ii)

$$v^2 = u^2 + 2as$$

$$196 = 0 + 2a(30)$$

$$a = \frac{196}{60} \text{ or } \frac{g}{3}$$

5

$$v = u + at$$

$$14 = 0 + \frac{196}{60}t$$

$$t = \frac{840}{196} = 4.29 \text{ seconds}$$

5

(iii)

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left\{mr^2\right\}\omega^2 = mg\{30\sin 30\}$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = 15mg$$

$$v^2 = 15g$$

5

$$v^2 = u^2 + 2as$$

$$15g = 0 + 2a(30)$$

$$a = \frac{147}{60} \text{ or } \frac{g}{4}$$

$$\frac{\text{acc. of hoop}}{\text{acc. of disc}} = \frac{\frac{1}{4}g}{\frac{1}{3}g} = \frac{3}{4}$$

5

30