8. (a) Prove that the moment of inertia of a uniform disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

Let
$$M = mass per unit area$$
 mass of element $= M\{2 \pi x.dx\}$ moment of inertia of the element $= M\{2 \pi x.dx\}x^2$ moment of inertia of the disc $= 2 \pi M \int_0^r x^3 dx$
$$= 2 \pi M \left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{1}{2} \pi M r^4$$

$$= \frac{1}{2} m r^2$$