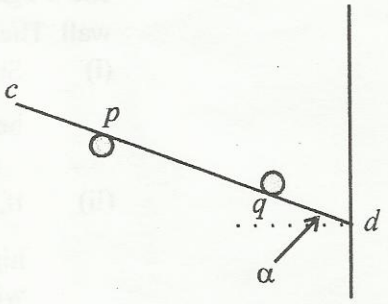
2000

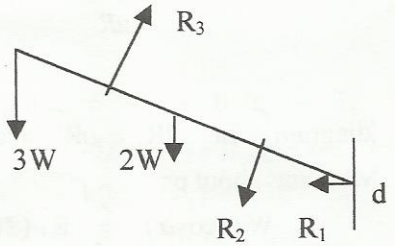
(b) A uniform rod $[cd]$ of weight $2W$ rests in equilibrium at an angle α to the horizontal with end d in contact with a smooth vertical wall. The rod passes over a smooth fixed peg at p and under a smooth fixed peg at q.
A weight of $3W$ is suspended from c.
$|cp| = |qd| = l$ and $|pq| = 2l$.



(i) Show that the magnitude of the reaction at p is $\frac{W}{2\cos\alpha}\left(16\cos^2\alpha - 5\right)$.

(ii) Find, in terms of α and W, the magnitude of the reaction at q.

(iii) Show that $\cos^2\alpha \geq \frac{15}{16}$.



(i)

vert: $R_3\cos\alpha = R_2\cos\alpha + 5W$ — 5

Moments about d:

$R_2(\ell) + 2W(2\ell\cos\alpha) + 3W(4\ell\cos\alpha) = R_3(3\ell)$ — 5

$\left(R_3 - \frac{5W}{\cos\alpha}\right) + 16W\cos\alpha = 3R_3$

$\Rightarrow \quad R_3 = \frac{W}{2\cos\alpha}\left\{16\cos^2\alpha - 5\right\}$ — 5

(ii)

$R_2 = 3R_3 - 16W\cos\alpha$

$= \frac{3W}{2\cos\alpha}\left(16\cos^2\alpha - 5\right) - 16W\cos\alpha$

$= \frac{W}{2\cos\alpha}\left(16\cos^2\alpha - 15\right)$ — 5

(iii)

$R_2 \geq 0 \quad \Rightarrow \quad 16\cos^2\alpha - 15 \geq 0$

$\cos^2\alpha \geq \frac{15}{16}$ — 5

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