2008

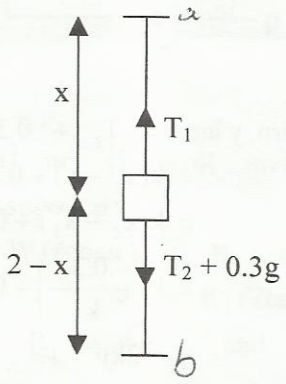
(b) A particle of mass 0.3 kg is attached to the midpoint of a light elastic string of natural length 1 m and elastic constant k. The string is then stretched between two points a and b. The point a is 2 m vertically above b.

Find

(i) the extensions of the two parts of the string, in terms of k, when the system is in equilibrium

(ii) the minimum value of k which will ensure that the lower part of the string is taut

(iii) the period of small oscillations, in terms of k, when the particle is displaced vertically. (Assume both parts of the string remain taut.)



(i) $T_1 = T_2 + 0.3g$

$k(x - \frac{1}{2}) = k(1\frac{1}{2} - x) + 0.3g$

$\Rightarrow \quad x = 1 + \dfrac{0.15g}{k}$

$x_1 = x - \frac{1}{2} = \dfrac{1}{2} + \dfrac{0.15g}{k}$ or $\dfrac{k + 0.3g}{2k}$

$x_2 = 1\frac{1}{2} - x = \dfrac{1}{2} - \dfrac{0.15g}{k}$ or $\dfrac{k - 0.3g}{2k}$

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(ii) $x_2 \geq 0 \quad \Rightarrow \quad k \geq 0.3g$

$\Rightarrow$ min value of $k = 0.3g$

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