2000 **(b)** The deceleration of a particle moving in a straight line with speed v m/s has magnitude $4e^{\frac{v}{6}}$ m/s^2. The particle has an initial speed of 6 m/s.

(i) Find the time t_1 for the speed to decrease to 3 m/s.

(ii) Find the time t_2 for the particle to come to rest.

(iii) Deduce that $\frac{t_2 - t_1}{t_1} = \sqrt{e}$.

(i)

$$\frac{dv}{dt} = -4e^{\frac{1}{6}V}$$

$$\int_6^3 e^{-\frac{1}{6}V}\,dv = -4\int_0^{t_1} dt$$

$$-6\left[e^{-\frac{1}{6}V}\right]_6^3 = -4t_1$$

$$t_1 = \frac{6}{4}\left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)$$

(ii)

$$-6\left[e^{-\frac{1}{6}V}\right]_6^0 = -4t_2$$

$$t_2 = \frac{6}{4}\left(1 - \frac{1}{e}\right)$$

(iii)

$$\frac{t_2 - t_1}{t_1} = \frac{\frac{6}{4}\left\{\left(1 - \frac{1}{e}\right) - \left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)\right\}}{\frac{6}{4}\left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)}$$

$$= \frac{1 - \frac{1}{\sqrt{e}}}{\frac{1}{\sqrt{e}} - \frac{1}{e}}$$

$$= \frac{e - \sqrt{e}}{\sqrt{e} - 1} = \frac{\sqrt{e}\left(\sqrt{e} - 1\right)}{\left(\sqrt{e} - 1\right)}$$

$$= \sqrt{e}$$

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