8 (a)   Prove that the moment of inertia of a uniform rod [ab] of mass m and length 2ℓ about an axis t through its centre perpendicular to the rod is $\frac{1}{3}m\ell^2$.

Let m_1 = mass per unit length

mass of element $= m_1 \, dx$

moment of inertia $= m_1 x^2 \, dx$

5

moment of inertia of rod $= \displaystyle\int_{-\ell}^{\ell} m_1 x^2 \, dx$

5

$= m_1 \left[\dfrac{x^3}{3} \right]_{-\ell}^{\ell}$

5

$= \dfrac{2}{3} m_1 \ell^3$

$= \dfrac{1}{3} m \ell^2$

5

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