

- 5 (b) Two equal smooth spheres A and B collide. The velocity of A before the collision is $3\sqrt{3} \vec{i} + 3 \vec{j}$ and the velocity of B before the collision is $\frac{1}{2}(-u\sqrt{3} \vec{i} + u \vec{j})$ where \vec{i} and \vec{j} are unit perpendicular vectors along and perpendicular to the line of centres, respectively. The velocity of A after the collision is $\frac{1}{2}(-v \vec{i} + v\sqrt{3} \vec{j})$. If the coefficient of restitution is 0.7, find the magnitude and direction of the velocity of sphere B after the collision.

	mass	velocity before	velocity after
A	m	$3\sqrt{3} \vec{i} + 3 \vec{j}$	$-\frac{v}{2} \vec{i} + \frac{v\sqrt{3}}{2} \vec{j}$
B	m	$-\frac{u\sqrt{3}}{2} \vec{i} + \frac{u}{2} \vec{j}$	$v_1 \vec{i} + \frac{u}{2} \vec{j}$

\vec{j} component of the velocity of A does not change

$$\Rightarrow \frac{v\sqrt{3}}{2} = 3 \quad \text{or} \quad v = 2\sqrt{3}$$

PCM(\vec{i} dirn) $m(3\sqrt{3}) + m\left(-\frac{u\sqrt{3}}{2}\right) = m\left(-\frac{v}{2}\right) + mv_1$ 5

$$3\sqrt{3} + \left(-\frac{u\sqrt{3}}{2}\right) = -\sqrt{3} + v_1$$

$$\Rightarrow v_1 = 4\sqrt{3} - \frac{u\sqrt{3}}{2}$$

NEL(\vec{i} dirn) $v_1 + \frac{v}{2} = -e\left(\frac{-u\sqrt{3}}{2} - 3\sqrt{3}\right)$ 5

$$4\sqrt{3} - \frac{u\sqrt{3}}{2} + \sqrt{3} = 0.7\left(\frac{u\sqrt{3}}{2} + 3\sqrt{3}\right)$$

$$2.9 = 0.85u$$

$$\Rightarrow u = \frac{2.9}{0.85} \quad \text{or} \quad 3.41 \quad \text{or} \quad \frac{58}{17}$$

$$\text{velocity of B} = \frac{39\sqrt{3}}{17} \vec{i} + \frac{29}{17} \vec{j} \quad \text{or} \quad 3.97 \vec{i} + 1.71 \vec{j}$$

$$\text{magnitude} = 4.32$$

$$\text{direction} \quad \theta = \tan^{-1}\left(\frac{29}{39\sqrt{3}}\right) \quad \text{or} \quad 23.23^\circ \quad \text{or} \quad 23^\circ 14'$$
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