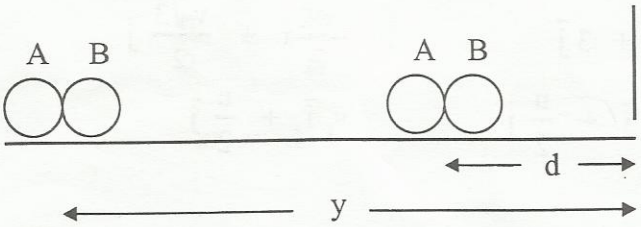
1999

5 (a) A smooth sphere moves on a horizontal table. It strikes an identical sphere at rest on the table. The latter is at a distance y from a vertical cushion. The impact is along the line of centres and normal to the cushion. The next collision between the spheres takes place at a distance d from the cushion.

(i) Prove that $d = \frac{2e^2 y}{1+e^2}$ where e is the coefficient of restitution for impacts between spheres and between a sphere and the cushion.

(ii) Interpret the result when $e = 1$.

1999 (a)



	mass	velocity before	velocity after
A	m	u	v_1
B	m	0	v_2

(i) PCM $\quad m(u) + m(0) = mv_1 + mv_2$ $\quad$ 5

NEL $\quad v_1 - v_2 = -e(u - 0)$ $\quad$ 5

$v_1 = \frac{u}{2}(1 - e)$ and $v_2 = \frac{u}{2}(1 + e)$ $\quad$ 5,5

Rebound velocity of B $= e v_2$ or $\frac{eu}{2}(1 + e)$ $\quad$ 5

Second collision

$$\frac{y-d}{v_1} = \frac{y}{v_2} + \frac{d}{ev_2}$$

$$\frac{y-d}{\frac{u}{2}(1-e)} = \frac{y}{\frac{u}{2}(1+e)} + \frac{d}{\frac{eu}{2}(1+e)}$$

$$\frac{y-d}{1-e} = \frac{ey+d}{e(1+e)}$$

$$ey - ed + e^2y - e^2d = ey + d - e^2y - ed$$

$$\Rightarrow \quad d = \frac{2ye^2}{1+e^2}$$ $\quad$ 5

(ii) $e = 1 \quad \Rightarrow \quad d = y$

A stops after the first collision and the second collision between the spheres occurs at a distance y from the cushion. $\quad$ 5