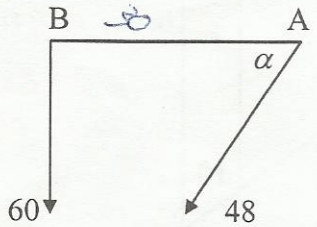
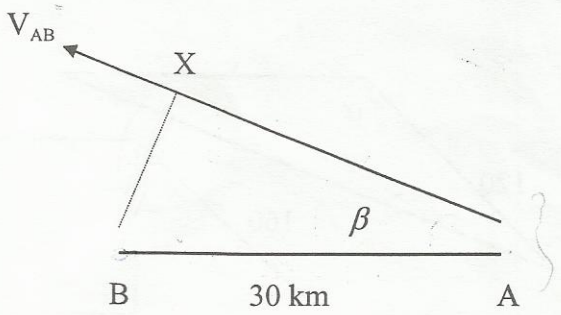
2000

2 (b) Two ships A and B move with constant speeds 48 km/h and 60 km/h respectively. At a certain instant ship B is 30 km west of A and is travelling due south. Find

(i) the direction ship A should steer in order to get as close as possible to ship B

(ii) the shortest distance between the ships.





(i) $V_{AB} = V_A - V_B$ (or diagram with V_{AB}) 5

$V_{AB} = (-48\cos\alpha\,\vec{i} - 48\sin\alpha\,\vec{j}) - (-60\vec{j})$ 5,5

$= (-48\cos\alpha)\vec{i} + (60 - 48\sin\alpha)\vec{j}$

$$\tan\beta = \left(\frac{V_{AB}\sin\beta}{V_{AB}\cos\beta}\right) = \frac{60 - 48\sin\alpha}{48\cos\alpha}$$

$$\tan\beta = \frac{5 - 4\sin\alpha}{4\cos\alpha}$$

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$$\frac{d(\tan\beta)}{d\alpha} = \frac{4\cos\alpha(-4\cos\alpha) - (5 - 4\sin\alpha)(-4\sin\alpha)}{(4\cos\alpha)^2}$$

$= 0$ when $16\cos^2\alpha = 20\sin\alpha - 16\sin^2\alpha$

$\Rightarrow \quad \sin\alpha = \frac{4}{5} \quad \left(\text{and } \tan\beta = \frac{3}{4}\right)$

(ii) $|BX| = 30\sin\beta$

$= 30\,(0.6)$

$= 18$ km 5