10 (b) The rocket engine of a 12 tonne missile produces a thrust of 180.1 kN. The missile is launched in a vertical direction. The air resistance is $v^2$ N where v is the speed of the missile.

      (i) Find the speed of the missile after 30 seconds.

      (ii) Find the percentage error in this speed if air resistance is ignored.

(i)                      Force = mass x acceleration

$$180100 - 12000\,g - v^2 = 12000\,\frac{dv}{dt}$$

5

$$62500 - v^2 = 12000\,\frac{dv}{dt}$$

$$\int_0^{30} dt = 12000 \int_0^v \frac{dv}{250^2 - v^2}$$

10

$$30 = (12000)\left(\frac{1}{500}\right) \ln\left|\frac{250 + v}{250 - v}\right|$$

$$\ln\left|\frac{250 + v}{250 - v}\right| = 1.25$$

$$v = 138.64$$

5

(ii)      Air resistance is omitted

$$62500 = 12000\,f$$

$$f = 5.2083$$

$$v = u + f\,t$$

$$= 0 + 5.2083\,(30)$$

$$= 156.25$$

$$\text{error} = 156.25 - 138.64 = 17.61$$

$$\text{Percentage error} = \frac{17.61 \times 100}{138.64} = 12.7\,\%$$

5