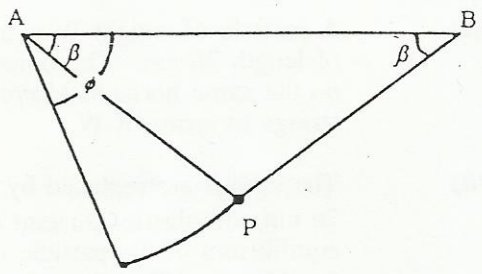
1997 NOTE No.6 = CIRCULAR MOTION 7(a) ~~7(b)~~ SHM 7(b) STATICS

6. A particle P, of mass m, is suspended by two inextensible light strings PA, PB, of equal length where A and B are fixed at the same horizontal level and each string is inclined at an angle β to the horizontal.

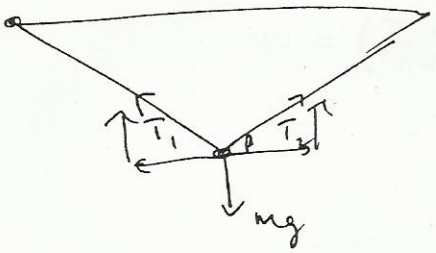


(i) Find the tension in string PA.

(ii) If the string PB is cut so that P starts to move in a circular path, prove that the tension in the string PA when it makes an angle ϕ with the horizontal is
$mg(3 \sin \phi - 2 \sin \beta)$.

(iii) If the tension in PA is suddenly halved when PB is cut, find the angle β.

(i)



By symmetry, The Tensions are equal

$T_1 = T_2 = T$.

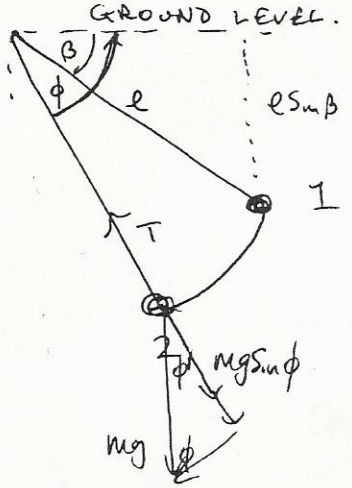
$T_1 \cos\beta = T_2 \cos\beta$

$T_1 = T_2$

$T_1 \sin\beta + T_2 \sin\beta = mg$

$2T \sin\beta = mg$

$T = \dfrac{mg}{2\sin\beta}$



Let ℓ = length of string:

$m g (-\ell \sin\beta) + \frac{1}{2} m (0)^2 = mg(-\ell \sin\phi) + \frac{1}{2} m v^2$

$$m v^2 = 2g\ell m(\sin\phi - \sin\beta)$$

$F_c = \dfrac{mv^2}{r}$

$T - mg\sin\phi = \dfrac{mv^2}{\ell}$

$$T\ell - mg\ell\sin\phi = mv^2$$

$2g\ell m \sin\phi - 2mg\ell \sin\beta = T\ell - mg\ell\sin\phi$

$3mg\sin\phi - 2mg\sin\beta = T$

$T = mg(3\sin\phi - 2\sin\beta)$

(iii) $T = mg(3\sin\beta - 2\sin\beta) = mg\sin\beta$

$mg\sin\beta = \frac{1}{2}\left(\dfrac{mg}{2\sin\beta}\right) \Rightarrow \sin^2\beta = \frac{1}{4} \Rightarrow \sin\beta = \frac{1}{2} \Rightarrow \beta = 60°$.