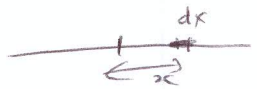
1993 8(a):



ρ = density

$\rho = \frac{dm}{dx} \Rightarrow dm = \rho dx$

and $\rho = \frac{m}{2\ell} \Rightarrow m = \rho 2\ell$

$dI = dm x^2$

$dI = \rho dx\, x^2$

$I = \int_{-\ell}^{\ell} \rho x^2 dx$

$= \rho \frac{x^3}{3} \Big|_{-\ell}^{\ell}$

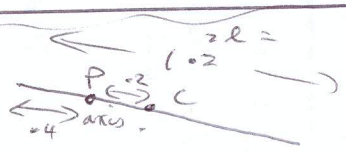
$= \rho \left[\frac{\ell^3}{3} - \frac{(-\ell)^3}{3} \right]$

$= \rho \frac{2\ell^3}{3}$

$= \frac{2\rho\ell\ell^2}{3}$

$= \frac{m\ell^2}{3}$ qed.

(i):

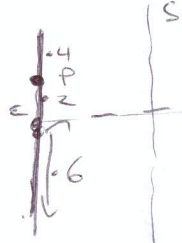


$I_P = I_C + m(PC)^2$

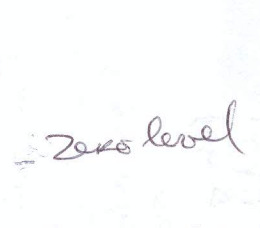
$= \frac{m(.6)^2}{3} + m(.2)^2$

$= 0.16m$

Sit 1



Sit 2



$v = .8w$ ASK

$\varepsilon_1 = 0 + \frac{1}{2} I \omega^2$

$= \frac{1}{2}(.16m)\omega^2$

$= .08m\omega^2$

Least value of v to reach top

$v \geq 0, \omega \geq 0$

$\varepsilon_2 = mg(.4) + 0$

$= mg(.4)$

PCE $\Rightarrow \varepsilon_1 = \varepsilon_2$

$\Rightarrow .08m\omega^2 = mg \cdot 4$

$\omega^2 = \frac{.4}{.08} g$

$\omega^2 = 5(9.8)$

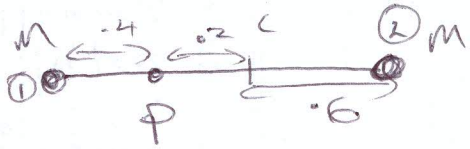
$\omega^2 = 49$

Least $\omega = 7, \Rightarrow \omega \geq 7$

$v = .8\omega \Rightarrow \frac{v}{.8} \geq 7$

$v \geq 5.6 ms^{-1}$.

(ii) $T = 2\pi \sqrt{\frac{I}{mgh}}$



$I_{P\,syst} = I_{P\,rod} + I_{P\,①} + I_{P\,②}$

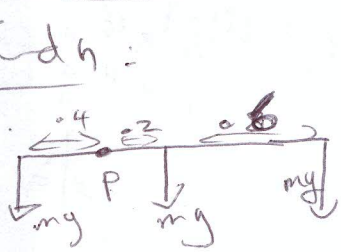
$= .16m + m(.4)^2 + m(.8)^2$

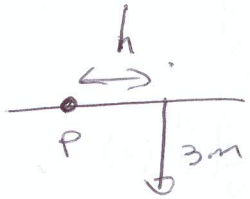
$= .16m + .16m + .64m$

$= .96m$.

Total mass $= m + m + m = 3m$.

Find h:





$\sum$ mom $= -.4mg + .2mg + .8mg$

$= .6mg$

mom $\varepsilon = 3mgh$

Principle of moments $\Rightarrow$

$.6mg = 3mgh$

$.2 = h$.

$\therefore T = 2\pi \sqrt{\frac{.96m}{3m\, g(.2)}}$

$= 2\pi \sqrt{\frac{.96}{.6(9.8)}}$

$= 2\pi \sqrt{\frac{.96}{5.88}}$

$= 2\pi \sqrt{.1632653}$

$= 2(3.14)(.404061)$

$= 2.5375$ seconds

$= 2.54$ seconds to 2 dec places.