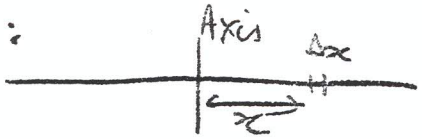
1988 RBM:



Density:

$\rho = \frac{m}{2\ell} \Rightarrow m = \rho 2\ell$

$\rho = \frac{\Delta m}{\Delta x} \Rightarrow \Delta m = \rho \Delta x$

$\Delta I = \Delta m x^2$

$= (\rho \Delta x) x^2$

$\Rightarrow I = \int_{-\ell}^{\ell} \rho x^2 dx$

$\Rightarrow I = \rho \left| \frac{x^3}{3} \right|_{-\ell}^{\ell}$

$\Rightarrow I = \frac{\rho}{3}\left(\ell^3 - (-\ell)^3\right)$

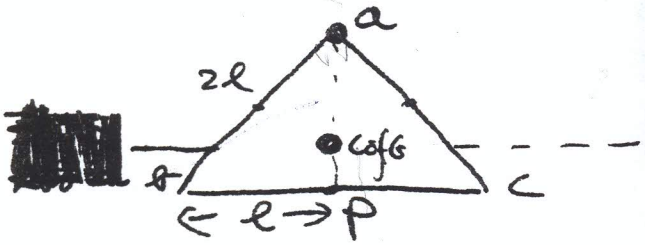
$\Rightarrow I = \frac{2\rho\ell^3}{3}$.

$\Rightarrow I = \frac{m}{3\ell}\ell^2(2\rho\ell)$

$\Rightarrow I = \frac{m\ell^2}{3}$

(b)

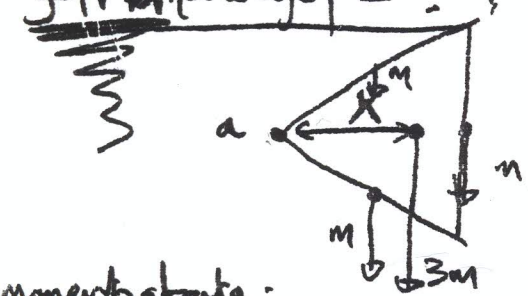
$$T = 2\pi\sqrt{\frac{I}{\text{mass}\, g\, (\text{dist to cofg})}}$$



$|ap|^2 = (2\ell)^2 - \ell^2 = 3\ell^2$

$\Rightarrow |ap| = \sqrt{3}\ell$

Find $|a\,\text{to cofG}| = ?$



moments about a:

$m\left(\frac{\sqrt{3}\ell}{2}\right) + m\frac{\sqrt{3}\ell}{2} + m\sqrt{3}\ell = 3mX$

$2\sqrt{3}\ell = 3X$

$\frac{2\sqrt{3}\ell}{3} = X$

MASS (Total) $= 3m$

Moments of Inertia

M of I of system about a.

(1) M of I of $[ab] = \frac{1}{3}m\ell^2$

(2) M of I of $[ac] = \frac{1}{3}m\ell^2$

(3) M of I of $[bc]$ about $p = \frac{1}{3}m\ell^2$.

$\Rightarrow$ M of I of $[bc]$ about a

$= \frac{1}{3}m\ell^2 + m|ap|^2$

$= \frac{1}{3}m\ell^2 + m(\sqrt{3}\ell)^2$.

$= \frac{1}{3}m\ell^2 + 3m\ell^2$

$= \frac{10}{3}m\ell^2$.

$\therefore$ Total M of I of $\triangle abc$ about a

$= \left(\frac{4}{3} + \frac{4}{3} + \frac{10}{3}\right)m\ell^2$

$= 6m\ell^2$.

$\therefore T = 2\pi\sqrt{\frac{6m\ell^2}{(3m)\, g\left(\frac{2\sqrt{3}\ell}{3}\right)}}$

$= 2\pi\sqrt{\frac{6m\ell^2}{2mg\sqrt{3}\ell}}$

$= 2\pi\sqrt{\frac{3\ell}{\sqrt{3}g}}$

$= 2\pi\sqrt{\frac{\sqrt{3}\ell}{g}}$.