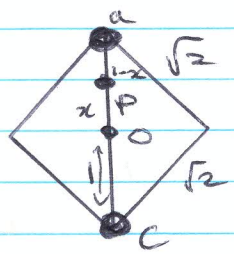
1985

Rigid Body Rotation. (Board)



Particles mass m at a and c
$|ac| = 2$ $|ao| = 1$ $|op| = 1-x$ $|po| = x$
c of G of lamina is at o
mass of lamina.
Total mass of system is $5m$

(i) Find I of system about P.

$$I = Im(1-x)^2 + m(1+x)^2 + I_{lamina}.$$

$I_{lamina} = I_o + (3m)|op|^2$ (parallel axes thm)
Now $I_o = I_x + I_y$ (by ⊥ axes thm)

$$I_x = \frac{1}{3}(3m)\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{3m}{3}\cdot\frac{2}{4} = \frac{1}{2}m$$

$I_y = \frac{1}{2}m$ since $I_x = I_y$ (square)

$\therefore I_o = m$

$\therefore I_{lamina} = m + 3m(x)^2$

$\therefore I = m(1-x)^2 + m(1+x)^2 + 3mx^2 + m$

$= m\,|\,-2mx + mx^2 + m + 2mx + mx^2 + 3mx^2 + m$

$= 3m + 5mx^2$

$= m(3+5x^2)$ qed

Small oscillations $\Rightarrow$ $T = 2\pi\sqrt{\dfrac{I}{(\text{mass})\,g\,(y)}}$

where mass $= 5m$

$I = m(3+5x^2)$

and y = distance from p to c of G of system.

Here c of G of system is at o. [By symmetry argument or
(a) $3m(1) + m(2) = 5m\bar{x}$
$\Rightarrow \bar{x} = 1$.
ie c of G at o]

$\Rightarrow \boxed{y = |op| = x}$

$$\Rightarrow T = 2\pi\sqrt{\frac{m(3+5x^2)}{5mgx}}$$

$$= 2\pi\sqrt{\frac{3+5x^2}{5gx}}$$

Remember if fraction = 0 numerator must be zero

T min for $x = ?$ $\quad \frac{dT}{dx} = 0$ at min.

$$\Rightarrow \frac{d}{dx}\left[2\pi\left(\frac{3+5x^2}{5gx}\right)^{\frac{1}{2}}\right] = 0 \Rightarrow 2\pi\tfrac{1}{2}\left(\frac{3+5x}{5gx}\right)^{-\frac{1}{2}}\left[\frac{5gx\,10x - (3+5x^2)5g}{(5gx)^2}\right] = 0$$

$$\Rightarrow 50gx^2 - 15g - 25gx^2 = 0 \Rightarrow 25gx^2 - 15g = 0$$

$$\Rightarrow x = \sqrt{0.6}$$