Eg(3) (1983)

(16)

8. ~~Define~~ simple harmonic motion in a straight line and show that

$$x = a \sin \omega t$$

can describe such motion, when x is the distance from a fixed point and a, ω and t have the usual meanings.

A particle p, of mass 5 kg, is connected by a light elastic string, of natural length 2 m and elastic constant 140 N/m to a fixed point q on a rough horizontal surface where the coefficient of friction is 1.
p is released from rest at a point a where $|qa| = 3$ m.
By considering the forces acting on p when its distance is $(2{\cdot}35 + x)$m from q, prove that p moves in simple harmonic motion as long as the string remains taut. State the position of the centre, o, of the simple harmonic motion i.e. $|qo|$ and write down the amplitude.

If the periodic time is assumed to be $\frac{\pi}{\sqrt{7}}$ calculate the time taken by the particle to travel from a to a point 2 m from q.

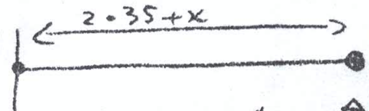
Def^n Notes.

$x = a \sin \omega t \Rightarrow \frac{dx}{dt} = a\omega \cos \omega t \Rightarrow \frac{d^2x}{dt^2} = -a\omega^2 \sin \omega t.$

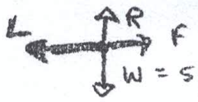
$\Rightarrow \text{accel} = -\omega^2(a \sin \omega t)$

$\boxed{a = \text{amplitude (care!)}}$ $\quad \Rightarrow \text{accel} = -\omega^2 x.$ q.e.d.

Typical position:



Forces:



Accel: a

Hookes Law: $L = +140(2{\cdot}35 + x - 2)$
$= +140(\cdot 35 + x)$
$= 49 + 140x$

Friction Law: $F = \mu R$
$\Rightarrow F = R. \ (\mu = 1)$

N II: $\sum \vec{F} = m\vec{a}$

$\vec{i}$ dir^n

$-L + F = 5a$

$-49 - 140x + R = 5a$

$-49 - 140x + 49 = 5a$

$\Rightarrow -140x = 5a$

$\boxed{-28x = a}$

$\vec{j}$ dir^n

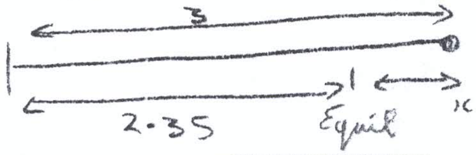
$R - W = 0.$

$R - 5(9{\cdot}8) = 0$

$R = 49$

Particle is performing SHM with $\omega = \sqrt{28}$ about a point 2·35 from wall.

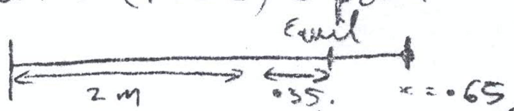
Find Amplitude: Told $v = 0$ where particle is 3m from wall.



$x = 3 - 2{\cdot}35$
$= \cdot 65.$

$v = 0$, where $x = \cdot 65 \Rightarrow \boxed{A = 0{\cdot}65}$

Find time to go from a (Extreme) to point 2m from wall



Want time for particle to go from the +Extreme position to a position where $x = -\cdot 35$

$x = A \cos \omega t$ (as starts at extreme)

At required position $x = -\cdot 35$: $\quad -\cdot 35 = \cdot 65 \cos \sqrt{28}t.$

$\Rightarrow -\frac{35}{65} = \cos \sqrt{28}t.$

$\Rightarrow -\cdot 5385 = \cos \sqrt{28}t \quad \Rightarrow 2{\cdot}139 = \sqrt{28}t$

$\Rightarrow 0{\cdot}404 \text{ secs} = t$