H.Spec I Q5

SHM about $x=0$, $\Rightarrow$ accel $= -\omega^2 x$.

when $x = 1\,m$ $\quad v = 4\sqrt{2}\,ms^{-1}$.

$v^2 = \omega^2(A^2 - x^2) \Rightarrow (4\sqrt{2})^2 = \omega^2(A^2 - 1^2)$

$32 = \omega^2(A^2 - 1)$ ①

when $x = 2$, $v = 2\sqrt{5}\,m$

$v^2 = \omega^2(A^2 - x^2) \Rightarrow (2\sqrt{5})^2 = \omega^2(A^2 - 2^2)$

$\Rightarrow 20 = \omega^2(A^2 - 4)$ ②

Solve ① and ②, ① ÷ ②

$\frac{32}{20} = \frac{A^2 - 1}{A^2 - 4}$.

$\Rightarrow 32(A^2 - 4) = 20(A^2 - 1)$

$\Rightarrow 32A^2 - 128 = 20A^2 - 20$

$\Rightarrow 12A^2 = 108$

$\Rightarrow A^2 = 9$

$\Rightarrow \boxed{A = \pm 3}$

Sol ① $\Rightarrow 32 = \omega^2(3^2 - 1)$

$32 = \omega^2 8$

$4 = \omega^2$

$\boxed{2 = \omega}$

$\therefore T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = \frac{2\pi}{2} = \pi \text{ secs}}$

Starting at centre $x = 0$, Find time to reach point where $v = 4\,ms^{-1}$.

step I: Find distance to reach such a point.

$v = 4, \omega = 2, A = 3$, $\quad v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow 4^2 = 2^2(3^2 - x^2)$

$\Rightarrow 16 = 4(9 - x^2)$

$\Rightarrow 4 = 9 - x^2$

$\Rightarrow -5 = -x^2$

$\Rightarrow 5 = x^2 \Rightarrow \boxed{x = \pm\sqrt{5}}$

So starting from centre we want to find time to reach $x = \sqrt{5}\,m$.

$x = A\sin\omega t \Rightarrow \sqrt{5} = 3\sin 2t$

$\Rightarrow \frac{\sqrt{5}}{3} = \sin 2t$

$\Rightarrow t = \frac{1}{2}\sin^{-1}\frac{\sqrt{5}}{3}$

$t = .421$ secs.

OR use $\boxed{v = A\omega\cos\omega t}$ ASK.

1971, Q5

accel $= -\omega^2 x$.

But accel $= \frac{dv}{dt} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d^2x}{dt^2}$.

$\Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x$. (*)

Show $x = a\cos\omega t$.

$\Rightarrow \frac{dx}{dt} = a\omega(-\sin\omega t)$

$\Rightarrow \frac{d^2x}{dt^2} = a\omega^2(-\cos\omega t)$

$= -\omega^2(a\cos\omega t)$

$\frac{d^2x}{dt^2} = -\omega^2 x$.

$\Rightarrow x = a\cos\omega t$ satisfies the D.E. (*).

when $v = 4$, $x = 1$

$v^2 = \omega^2(A^2 - x^2) \Rightarrow 4^2 = \omega^2(A^2 - 1)$

$\Rightarrow 16 = \omega^2(A^2 - 1)$ ①

when $v = 2$, $x = 2$

$v^2 = \omega^2(A^2 - x^2) \Rightarrow 2^2 = \omega^2(A^2 - 2^2)$

$\Rightarrow 4 = \omega^2(A^2 - 4)$ ②

$\frac{①}{②} \Rightarrow \frac{16}{4} = \frac{A^2 - 1}{A^2 - 4} \Rightarrow 16(A^2 - 4) = 4(A^2 - 1)$

$\Rightarrow 16A^2 - 64 = 4A^2 - 4$.

$\Rightarrow 12A^2 = 60$

$\Rightarrow A^2 = 5$

$\Rightarrow A = \pm\sqrt{5}$.

Find ω:

① $\Rightarrow 16 = \omega^2\left((\sqrt{5})^2 - 1^2\right)$

$\Rightarrow 16 = \omega^2(5 - 1)$

$\Rightarrow 16 = \omega^2 4 \Rightarrow \omega^2 = 4 \Rightarrow \boxed{\omega = 2}$

Find T: $T = \frac{2\pi}{\omega} \Rightarrow T = \pi$ seconds

Find the time to reach position where $v = 2\,ms^{-1}$ from a position of rest

Position of rest is at extreme position.

step I: Find displacement from centre where $v = 2\,ms^{-1}$.

$v^2 = \omega^2(A^2 - x^2) \to 2^2 = 2^2\left((\sqrt{5})^2 - x^2\right)$

$\Rightarrow 1 = 5 - x^2$

$\Rightarrow x = \pm 2$ metres.

step II: Find time to go from extreme position $x = \sqrt{5}$ metre to place where $x = 2\,m$

$x = A\cos\omega t$ (Extreme start)

$2 = \sqrt{5}\cos 2t$.

$\Rightarrow t = \frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}$

$\Rightarrow \boxed{t = .229}$ OR use $v = A\omega\sin\omega t$.