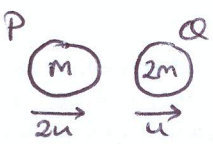
2009 HL

5. **(a)** A smooth sphere P, of mass m kg, moving with speed $2u$ m/s collides directly with a smooth sphere Q, of mass $2m$ kg, moving in the same direction with speed u m/s.
The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) Prove that the speed of Q increases after the collision.

(iii) Find the value of e if the speed of P after the collision is $\frac{10u}{9}$ m/s.



	Before	Mass	After
P	$u_1 = 2u$	$M_1 = m$	v_1
Q	$u_2 = u$	$M_2 = 2m$	v_2

$M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$

(i) PCM $\quad m(2u) + 2m(u) = mv_1 + 2mv_2 \quad$ ① $\quad$ 5

$v_1 - v_2 = -e(u_1 - u_2)$

NEL $\quad v_1 - v_2 = -e(2u - u) \quad$ ② $\quad$ 5

Solving ① + ②

÷m ① $\quad 4u = v_1 + 2v_2$

−1 × ② $\quad eu = -v_1 + v_2$

$(4+e)u = 3v_2$

$\frac{u(4+e)}{3} = v_2$

Sub into ②

$v_1 = \frac{u(4+e)}{3} - ue$

$= \frac{4u + ue - 3ue}{3}$

$= \frac{4u - 2ue}{3} = \frac{u(4-2e)}{3}$

$\boxed{v_1 = \frac{u(4-2e)}{3}}$ $\quad$ 5

$\boxed{v_2 = \frac{u(4+e)}{3}}$ $\quad$ 5

(ii) $\quad u_2 = u \qquad v_2 = \frac{u(4+e)}{3}$

which is $> u$ as $e > 0$

$\Rightarrow$ speed of Q increases $\quad$ 5

(iii) $\quad v_1 = \frac{u(4-2e)}{3}$

$\frac{10u}{9} = \frac{u(4-2e)}{3}$

$\frac{10u}{9} = \frac{u(4-2e)}{3} \div u \times 9$

$10 = 3(4-2e)$

$10 = 12 - 6e$

$-2 = -6e$

$\frac{-2}{-6} = e$

$\frac{1}{3} = e$

$\Rightarrow e = \frac{1}{3}$ $\quad$ 5 $\quad$ **30**