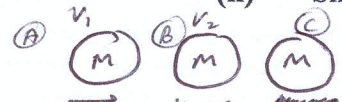
2008 HL

5. (a) Three identical smooth spheres lie at rest on a smooth horizontal table with their centres in a straight line. The first sphere is given a speed 2 m/s and it collides directly with the second sphere. The second sphere then collides directly with the third sphere.
The coefficient of restitution for each collision is e, where $e < 1$.

(i) Find, in terms of e, the speed of each sphere after two collisions have taken place.

(ii) Show that there will be at least one more collision.



1st and 2nd sphere

(*i*) PCM $m(2) + m(0) = mv_1 + mv_2$ ① 5

NEL $v_1 - v_2 = -e(2-0)$ ②

$m \div$ ① $v_1 + v_2 = 2$
② $v_1 - v_2 = -2e$
$2v_1 = 2 - 2e$
$v_1 = 1 - e$.
$v_2 = 2 - (1-e)$
$= 1 + e$

$v_1 = 1-e$ and $v_2 = 1+e$ 5

2nd + 3rd sphere

PCM $m(1+e) + m(0) = mv_3 + mv_4$ 5

NEL $v_3 - v_4 = -e(1+e)$

$m \div$ ① $v_3 + v_4 = 1 + e$
② $v_3 - v_4 = -e - e^2$
$2v_3 = 1 - e^2$
$v_3 = \frac{1}{2}(1-e^2)$.

$v_4 = 1 + e - \frac{1}{2}(1-e^2)$
$= 1/2 + e + \frac{1}{2}e^2$
$= \frac{1}{2}(1+e)^2$

$v_3 = \frac{1}{2}(1-e^2)$ and $v_4 = \frac{1}{2}(1+e)^2$ 5

speeds after 2nd impact : A $1-e$, B $\frac{1}{2}(1-e^2)$, C $\frac{1}{2}(1+e)^2$

(*ii*) First sphere will collide : again with second sphere if

$$1 - e > \tfrac{1}{2}(1-e^2)$$

$$1 - e > \tfrac{1}{2} - \tfrac{1}{2}e^2$$

$$e^2 - 2e + 1 > 0$$

$$(e-1)^2 > 0$$

This is true for $e < 1$ 5 | 25

Second sphere will collide with 3rd if
$\frac{1}{2}(1-e^2) > \frac{1}{2}(1+e)^2$
if $1 - e^2 > 1 + 2e + e^2$
$0 > 2e + 2e^2$
$0 > 2e(1+e)$ which is not true as
$e > 0 \therefore 2e(1+e) > 0$.

$\therefore$ 1st sphere will collide again with second sphere
$\therefore$ there is at least one more collision