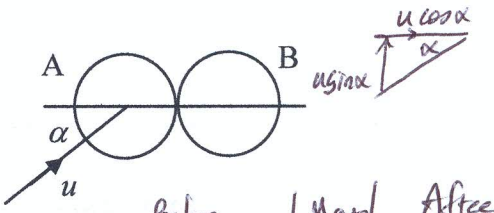
2007 HL

5 **(b)** A smooth sphere A, of mass 4 kg, moving with speed u, collides with a stationary smooth sphere B of mass 8 kg.
The direction of motion of A, before impact, makes an angle α with the line of centres at impact.



The coefficient of restitution between the spheres is $\frac{1}{2}$ ($e = \frac{1}{2}$).

	Before	Mass	After
A	$u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$	4kg	$v_1\vec{i} + u\sin\alpha\,\vec{j}$
B	$0\vec{i} + 0\vec{j}$	8kg	$v_2\vec{i} + 0\vec{j}$

Collision in $\vec{i}$ dir ⇒ no change in $\vec{j}$ vel.

Find in terms of u and α

(i) the speed (not velocity!) of each sphere after the collision
(ii) the angle through which the 4 kg sphere is deflected as a result of the collision
(iii) the loss in kinetic energy due to the collision.

(i) PCM ($\vec{i}$ dir) $\quad 4(u\cos\alpha) + 8(0) = 4v_1 + 8v_2$ ①

NEL ($\vec{i}$ dir) $\quad v_1 - v_2 = -\frac{1}{2}(u\cos\alpha - 0)$ ②

$+$ ① $4v_1 + 8v_2 = 4u\cos\alpha$
$-\times$ ② $-v_1 + v_2 = +\frac{1}{2}u\cos\alpha$
$3v_2 = \frac{3}{2}u\cos\alpha$
$v_2 = \frac{1}{2}u\cos\alpha$
Sub into ②
$v_1 = -\frac{1}{2}u\cos\alpha + \frac{1}{2}u\cos\alpha = 0$

$\Rightarrow \quad v_1 = 0$ and $v_2 = \frac{1}{2}u\cos\alpha$

Speed of A $= u\sin\alpha$ (Vel of A $= 0\vec{i} + u\sin\alpha\,\vec{j}$)
Speed of B $= \frac{1}{2}u\cos\alpha$ (Vel of B $= \frac{1}{2}u\cos\alpha\,\vec{i} + 0\vec{j}$)

(ii) Angle $= 90 - \alpha$

(iii) KE before $= \frac{1}{2}(4)u^2 = 2u^2$ $\left(\frac{1}{2}(4)u^2 + \frac{1}{2}(8)0^2\right)$

KE after $= \frac{1}{2}(4)\{u\sin\alpha\}^2 + \frac{1}{2}(8)\left\{\frac{1}{2}u\cos\alpha\right\}^2$
$= 2u^2\sin^2\alpha + u^2\cos^2\alpha$

$\frac{1}{2}(4)(v_1)^2 + \frac{1}{2}(8)(v_2)^2$
$= 2(u\sin\alpha)^2 + 4\left(\frac{1}{2}u\cos\alpha\right)^2$
$2u^2\sin^2\alpha + u^2\cos^2\alpha$

Loss in KE $= 2u^2 - 2u^2\sin^2\alpha - u^2\cos^2\alpha$
$= 2u^2(1 - \sin^2\alpha) - u^2\cos^2\alpha$
$= u^2\cos^2\alpha$

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