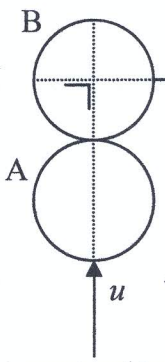
2006 HL

5 **(b)** A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is moving in a perpendicular direction with the same speed u.



let masses be m.

Before	Mom	After
(A) $0\vec{i}+u\vec{j}$	m	$0\vec{i}+v_1\vec{j}$
(B) $u\vec{i}+0\vec{j}$	m	$u\vec{i}+v_2\vec{j}$

The line of centres at the instant of impact is perpendicular to the direction of motion of sphere B.

The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after impact and hence, or otherwise, show that it is not possible for the two spheres to have the same speed after impact.

collision along $\vec{j}$ axis ⇒ no change in velocity in $\vec{i}$ coord.

(ii) Prove that $\tan\theta = \left(\frac{1+e}{2}\right)$, where θ is the angle through which sphere B is turned as a result of the impact.

($\vec{j}$ dir)

(i)

PCM $\quad m(u) + m(0) = mv_1 + mv_2$ ①

NEL $\quad v_1 - v_2 = -e(u-0)$ ②

① ÷ m $\quad u = v_1 + v_2$

② $\quad -eu = v_1 - v_2$

$u - eu = 2v_1$

$\frac{1}{2}u(1-e) = v_1$

sub into ②

$\frac{1}{2}u(1-e) + eu = v_2$

$\frac{1}{2}u - \frac{1}{2}eu + eu = v_2$

$\frac{1}{2}u + \frac{1}{2}eu = v_2$

$\frac{1}{2}u(1+e) = v_2$

$\Rightarrow \quad v_1 = \frac{u}{2}(1-e) \quad \text{and} \quad v_2 = \frac{u}{2}(1+e)$ — 5

Velocity of A $= 0\vec{i} + \frac{u}{2}(1-e)\vec{j}$. Vel of B $= u\vec{i} + \frac{u}{2}(1+e)\vec{j}$.

Speed of A $= \frac{u}{2}(1-e)$

Speed of B $= \sqrt{u^2 + \frac{u^2}{4}(1+e)^2}$ or $\frac{u}{2}\sqrt{e^2+2e+5}$ — 5

If Speed of A = Speed of B

then $\frac{u}{2}(1-e) = \frac{u}{2}\sqrt{e^2+2e+5}$

$\Rightarrow \quad e = -1$

$\Rightarrow$ The speeds can never be equal — 5

or

$\frac{u}{2}(1-e) = \sqrt{u^2 + \frac{u^2}{4}(1+e)^2}$

$\frac{u^2}{4}(1-e)^2 = u^2 + \frac{u^2}{4}(1+e)^2$ ×4

$u^2(1-2e+e^2) = 4u^2 + u^2(1+2e+e^2)$

$u^2 - 2eu^2 + u^2e^2 = 4u^2 + u^2 + 2eu^2 + u^2e^2$

$4u^2 + 4eu^2 = 0$

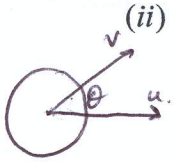
$4u^2(1+e) = 0$

$4u^2 = 0$ or $1+e = 0$

$u = 0$ means no initial movement $e = -1$ but $e > 0$.

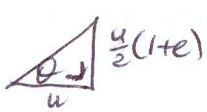
∴ speeds not equal.

(ii)



Velocity of B after the collision is

$u\vec{i} + \frac{u}{2}(1+e)\vec{j}$



$\tan\theta = \frac{\frac{u}{2}(1+e)}{u} = \frac{1}{2}(1+e)$ — 5

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