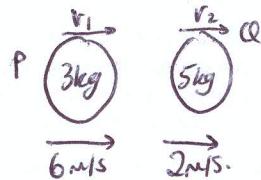


2006 HL

5. (a)

A smooth sphere P, of mass 3 kg, moving with speed 6 m/s, collides directly with a smooth sphere Q, of mass 5 kg, which is moving in the same direction with speed 2 m/s. The coefficient of restitution for the collision is e .

- Find, in terms of e , the speed of each sphere after the collision.
- If the loss of kinetic energy due to the collision is $k(1 - e^2)$, find the value of k .



$$(i) \text{ PCM} \quad 3(6) + 5(2) = 3v_1 + 5v_2 \quad (1)$$

$$\begin{aligned} (1) \quad 3v_1 + 5v_2 &= 28 \\ (2) \quad 5v_1 - 5v_2 &= -20e \end{aligned}$$

$$8v_1 = 28 - 20e$$

$$v_1 = \frac{28 - 20e}{8}$$

$$v_1 = \frac{7 - 5e}{2}$$

sub into (2)

$$\frac{7 - 5e}{2} + 4e = v_2$$

$$\frac{7 - 5e + 8e}{2} = v_2$$

$$\frac{7 + 3e}{2} = v_2$$

$$v_1 - v_2 = -e(6 - 2) \quad (2)$$

$$\left. \begin{aligned} v_1 &= \frac{7 - 5e}{2} \\ v_2 &= \frac{7 + 3e}{2} \end{aligned} \right\}$$

$$\frac{1}{2}M_1u_1^2 + \frac{1}{2}M_2u_2^2$$

$$\text{KE before} = \frac{1}{2}(3)(6)^2 + \frac{1}{2}(5)(2)^2 = 64$$

$$\frac{1}{2}M_1v_1^2 + \frac{1}{2}M_2v_2^2$$

$$\text{KE after} = \frac{1}{2}(3)(v_1)^2 + \frac{1}{2}(5)(v_2)^2$$

$$\frac{1}{2}(3)\left(\frac{7 - 5e}{2}\right)^2 + \frac{1}{2}(5)\left(\frac{7 + 3e}{2}\right)^2$$

$$\frac{3}{8}(49 - 70e + 25e^2) + \frac{5}{8}(49 + 42e + 9e^2)$$

$$\text{Loss in KE} = 64 - \frac{3}{2}\left(\frac{7 - 5e}{2}\right)^2 - \frac{5}{2}\left(\frac{7 + 3e}{2}\right)^2$$

$$= 15(1 - e^2)$$

$$\Rightarrow k = 15$$

5

5

5

5

5

25