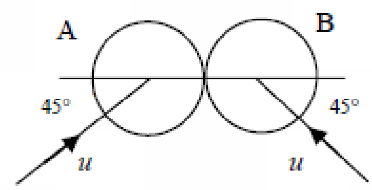
2005 – IMPACTS AND COLLISIONS QUESTION

5. **(a)** Three identical smooth spheres P, Q and R, lie at rest on a smooth horizontal table with their centres in a straight line. Q is between P and R.
Sphere P is projected towards Q with speed 2 m/s. Sphere P collides directly with Q and then Q collides directly with R.

The coefficient of restitution for all of the collisions is $\frac{3}{4}$.

Show that P strikes Q a second time.

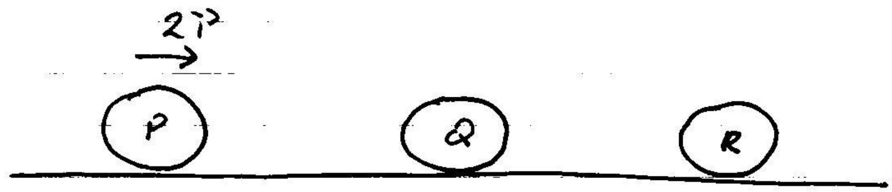
(b) A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B moving with speed u.
The direction of motion of A, before impact, makes an angle $45°$ with the line of centres at impact.
The direction of motion of B, before impact, makes an angle $45°$ with the line of centres at impact.
The coefficient of restitution between the spheres is e.



(i) Find, in terms of e and u, the speed of each sphere after the collision.

(ii) If $e = \frac{1}{2}$, show that after the collision the angle between the directions of motion of the two spheres is $\tan^{-1}\left(\frac{4}{3}\right)$.

Q5/(c)



$e = \frac{3}{4}$

$P \to Q$

Before	Mass	After
$2\vec{i}$	m	$p\vec{i}$
$0\vec{i}$	m	$q\vec{i}$

Con. of. Mom:

$(2)(m) + (0)(m) = (p)(m) + (q)(m)$

$\boxed{2 = p + q}$

Coeff. of Res:

$\frac{p - q}{2 - 0} = -\frac{3}{4}$

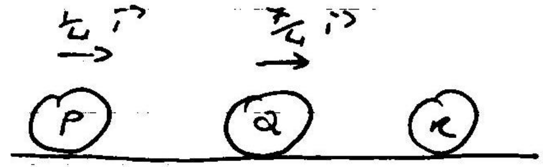
$p - q = -\frac{6}{4}$ ... $4p - 4q = -6$ ... $\boxed{2p - 2q = -3}$

Solving:

$$\begin{aligned} 2p - 2q &= -3 \\ 2p + 2q &= 4 \\ \hline 4p &= 1 \\ p &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} p + q &= 2 \\ \tfrac{1}{4} + q &= 2 \\ q &= 2 - \tfrac{1}{4} \\ q &= \frac{7}{4} \end{aligned}$$

Q5 (c) AFTER 1st COLLISION:



<u>$Q \to R$:</u>

	Before	Mass	After
	$\frac{7}{4}\vec{i}$	m	$s\vec{i}$
	$0\vec{i}$	m	$t\vec{i}$

<u>Con. of Mom:</u>

$$\frac{7}{4}(m) + 0(m) = s(m) + t(m)$$

$$\boxed{7 = 4s + 4t}$$

<u>Co-eff. of Res:</u>

$$\frac{s - t}{\frac{7}{4} - 0} = -\frac{3}{4}$$

$$s - t = \frac{7}{4} \times -\frac{3}{4}$$

$$\boxed{16s - 16t = -21}$$

Solving:

$$16s - \cancel{16t} = -21$$
$$16s + \cancel{16t} = 28$$
$$32s = 7$$
$$s = \frac{7}{32}$$

Vel. of P $= \frac{1}{4}\vec{i} = 0.25\vec{i}$

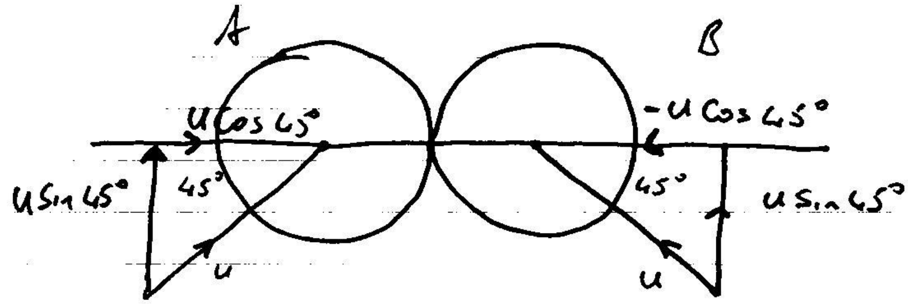
Vel of Q $= \frac{7}{32}\vec{i} = 0.21875\vec{i}$

Since

Vel of P $>$ Vel. of Q

P will strike Q a 2nd time

Q5 (b)



(i) **Before** — **Mass** — **After**

A: $u\cos 45°\,\vec{i} + u\sin 45°\,\vec{j}$ — m

$\frac{u}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$ — $p\,\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$

B: $-u\cos 45°\,\vec{i} + u\sin 45°\,\vec{j}$ — m

$-\frac{u}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$ — $q\,\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$

Con. of Mom:

$$\left(\frac{u}{\sqrt{2}}\right)(m) + \left(-\frac{u}{\sqrt{2}}\right)(m) = p(m) + q(m)$$

$$\frac{u}{\sqrt{2}} - \frac{u}{\sqrt{2}} = p + q$$

$$\boxed{0 = p + q}$$

Co-eff. of Res.:

$$\frac{p - q}{\frac{u}{\sqrt{2}} - \left(-\frac{u}{\sqrt{2}}\right)} = -e.$$

$$\frac{p-q}{\frac{2u}{\sqrt{2}}} = -e \quad \ldots \quad \boxed{p-q = -\frac{2ue}{\sqrt{2}}}$$

SOLVING:

$$\begin{aligned} p + q &= 0 \\ p - q &= -\frac{2ue}{\sqrt{2}} \\ \hline 2p &= -\frac{2ue}{\sqrt{2}} \quad \ldots \quad p = -\frac{ue}{\sqrt{2}} \end{aligned}$$

(i) $p + q = 0$

$$p = -q$$

so, $-\frac{ue}{\sqrt{2}} = -q \quad \ldots \quad q = \frac{ue}{\sqrt{2}}$

SPEED OF 1^{st} SPHERE: $-\frac{ue}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$

$$\text{Speed} = \sqrt{\left(\frac{-ue}{\sqrt{2}}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{u^2e^2}{2} + \frac{u^2}{2}}$$

$$= \sqrt{\frac{u^2}{2}}\sqrt{e^2+1}$$

so, SPEED OF 1^{st} SPHERE $= \frac{u}{\sqrt{2}}\sqrt{e^2+1}$

Speed of 2^{nd} sphere : $\frac{ue}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$

$$\text{Speed} = \sqrt{\left(\frac{ue}{\sqrt{2}}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{u^2e^2}{2} + \frac{u^2}{2}}$$

$$= \sqrt{\frac{u^2}{2}}\sqrt{e^2+1}$$

So, speed of 2^{nd} sphere $= \underline{\underline{\frac{u}{\sqrt{2}}\sqrt{e^2+1}}}$

(ii) If $e = \frac{1}{2}$

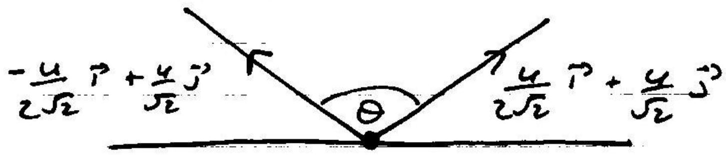
Vel. of 1^{st} : $\frac{-u\left(\frac{1}{2}\right)}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$

$$= \frac{-u}{2\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$$

Vel. of 2^{nd} : $\frac{u\left(\frac{1}{2}\right)}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$

$$= \frac{u}{2\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$$

After collision, motion of spheres is:



Slope of 1st: $\frac{\vec{j}}{\vec{i}} = \frac{\frac{u}{\sqrt{2}}}{-\frac{u}{2\sqrt{2}}} = \frac{u}{\sqrt{2}} \times \frac{2\sqrt{2}}{-u} = -2 = m_1$

Slope of 2nd: $\frac{\vec{j}}{\vec{i}} = \frac{\frac{u}{\sqrt{2}}}{\frac{u}{2\sqrt{2}}} = \frac{u}{\sqrt{2}} \times \frac{2\sqrt{2}}{u} = 2 = m_2$

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan\theta = \left| \frac{-2 - 2}{1 + (-2)(2)} \right|$$

$$\tan\theta = \left| \frac{-4}{1 - 4} \right|$$

$$\tan\theta = \left| \frac{-4}{-3} \right| \quad \ldots \quad \underline{\underline{\tan\theta = \frac{4}{3}}}$$