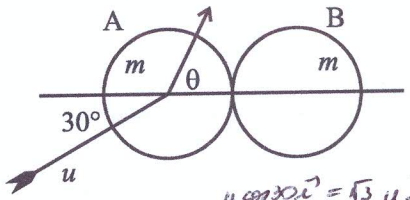
2004 HL

(b) A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest.
The direction of motion of A, before impact, makes an angle 30° with the line of centres at impact.
After impact the direction of A makes an angle θ with the line of centres, where $0° \le \theta < 90°$.
The coefficient of restitution between the spheres is e.
The **speeds** of A and B immediately after impact are equal. $v_1 = v_2$.



$u\cos 30\,\vec{i} = \frac{\sqrt{3}}{2}u\vec{i}$.
$u\sin 30\,\vec{j}$, $\frac{1}{2}u\vec{j}$, 30°, u

But not velocities.

(i) Calculate the value of θ.
(ii) Find e.

	Before	Mass	After
A	$\frac{\sqrt{3}}{2}u\vec{i} + \frac{1}{2}u\vec{j}$	m	$v_1\vec{i} + \frac{1}{2}u\vec{j}$
B	$0\vec{i} + 0\vec{j}$	m	$v_2\vec{i} + 0\vec{j}$.

PCM ($\vec{i}$ dirn) $\quad mu\cos 30 + m(0) = mv_1 + mv_2$ (÷m)
$\frac{\sqrt{3}}{2}u = v_1 + v_2$ ① — 5

Collision in $\vec{i}$-dirn ⇒ no change in $\vec{j}$-velocity.

NEL ($\vec{i}$ dirn) $\quad v_1 - v_2 = -e(u\cos 30 - 0)$
$v_1 - v_2 = -e\frac{u\sqrt{3}}{2}$ ② — 5

① $v_1 + v_2 = \frac{\sqrt{3}}{2}u$
② $v_1 - v_2 = -e\frac{u\sqrt{3}}{2}$
$2v_1 = \frac{\sqrt{3}u}{2}(1-e)$
$v_1 = \frac{\sqrt{3}u}{4}(1-e)$.

⇒ sub into ①
$v_2 = \frac{\sqrt{3}}{2}u - v_1$
$= \frac{\sqrt{3}}{2}u - \frac{\sqrt{3}}{4}u(1-e)$
$= \frac{2\sqrt{3}u - \sqrt{3}u + \sqrt{3}ue}{4}$
$v_2 = \frac{\sqrt{3}u(1+e)}{4}$

$$v_1 = \frac{u\sqrt{3}\{1-e\}}{4}$$ — 5

$$v_2 = \frac{u\sqrt{3}\{1+e\}}{4}$$

[Vel of A $= \frac{u\sqrt{3}(1-e)}{4}\vec{i} + \frac{u}{2}\vec{j}$] [Vel of B $= \frac{u\sqrt{3}(1+e)}{4}\vec{i} + 0\vec{j}$]

(ii) speed of A = speed of B.

$$(v_1)^2 + (u\sin 30)^2 = (v_2)^2$$

$$\frac{3u^2}{16}(1-2e+e^2) + \frac{u^2}{4} = \frac{3u^2}{16}(1+2e+e^2) \quad (\times \tfrac{16}{3u^2})$$

$1 - 2e + e^2 + 4/3 = 1 + 2e + e^2$
$4e = 4/3$
$\Rightarrow e = \frac{1}{3}$ — 5

(i) Vel of A $= \frac{u\sqrt{3}(1-e)}{4}\vec{i} + \frac{u}{2}\vec{j}$

$\frac{u}{2}$, $\frac{u\sqrt{3}}{4}(1-e)$, θ

$\tan\theta = \frac{\frac{u}{2}}{\frac{u\sqrt{3}(1-e)}{4}}$
$= \frac{u}{2} \cdot \frac{4}{u\sqrt{3}(1-e)}$

$$\tan\theta = \frac{\frac{u}{2}}{\frac{u\sqrt{3}\{1-e\}}{4}}$$

$$= \frac{2}{\sqrt{3}\{1-e\}}$$ but $e = \frac{1}{3}$.

$= \sqrt{3}$ $\left(\frac{2}{\sqrt{3}(\frac{2}{3})}\right)$
$\Rightarrow \theta = \tan^{-1}\sqrt{3}$
$\Rightarrow \theta = 60°$ — 5

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