2004 – IMPACTS AND COLLISIONS QUESTION

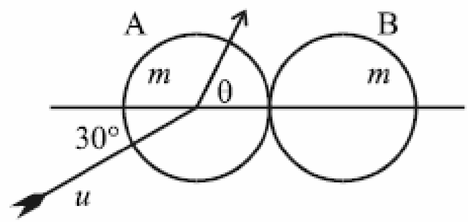
5. **(a)** A smooth sphere P, of mass $3m$, moving with speed u, collides directly with a smooth sphere Q, of mass $5m$, which is at rest.
The coefficient of restitution for the collision is e.

Find

(i) the speed, in terms of u and e, of each sphere after the collision

(ii) the condition to be satisified by e in order that the spheres move in opposite directions after the collision.

(b) A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest.
The direction of motion of A, before impact, makes an angle 30° with the line of centres at impact.



After impact the direction of A makes an angle θ with the line of centres, where $0° \leq \theta < 90°$.
The coefficient of restitution between the spheres is e.
The **speeds** of A and B immediately after impact are equal.

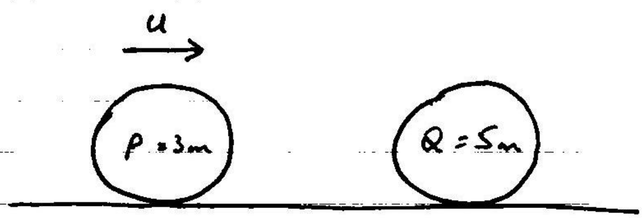
(i) Calculate the value of θ.

(ii) Find e.

Q5

(a)

(i)



BEFORE	MASS	AFTER
$u\vec{i}$	$3m$	$p\vec{i}$
$0\vec{i}$	$5m$	$q\vec{i}$

Con. of Mom.:

$$3m(u) + 5m(0) = 3m(p) + 5m(q)$$

$$\boxed{3u = 3p + 5q}$$

Coeff. of Res:

$$\frac{p - q}{u - 0} = -e$$

$$\boxed{p - q = -eu}$$

Solving:

$$3p + 5q = 3u$$
$$5p - 5q = -5eu$$

$$8p = 3u - 5eu$$

$$p = \frac{3u - 5eu}{8} \quad \ldots \quad p = \frac{u}{8}(3 - 5e)$$

Q5

(a) $p - q = -eu$

$p + eu = q$

so,

$$\frac{3u - 5eu}{8} + eu = q$$

$$\frac{3u - 5eu + 8eu}{8} = q \quad \ldots \quad q = \frac{3u + 3eu}{8}$$

(ii) SINCE $u > 0$ AND $e > 0$ THEN $q > 0$

AND SO IN ORDER FOR THE SPHERES TO MOVE IN OPPOSITE DIRECTIONS AFTER THE COLLISION $p < 0$

$$\therefore \quad \frac{3u - 5eu}{8} < 0 \quad (\div u)$$

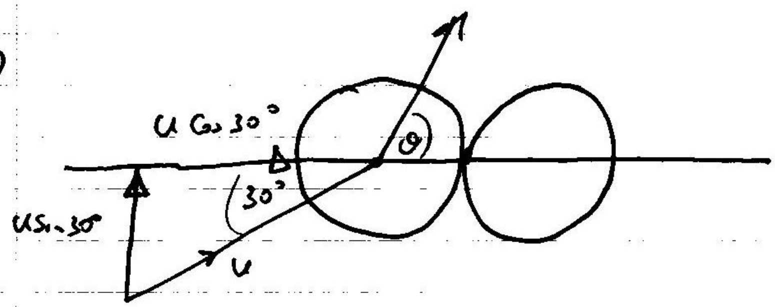
$$\frac{3 - 5e}{8} < 0 \quad (\times 8)$$

$$3 - 5e < 0$$

$$3 < 5e$$

$$\frac{3}{5} < e$$

Q5 (b)



BEFORE	MASS	AFTER
① $u\cos 30^\circ \vec{i} + u\sin 30^\circ \vec{j}$	m	$p\vec{i} + u\sin 30^\circ \vec{j}$
$\Rightarrow \frac{u\sqrt{3}}{2}\vec{i} + \frac{u}{2}\vec{j}$		$p\vec{i} + \frac{u}{2}\vec{j}$
② $0\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$

Con. of Mom:

$$\left(\frac{u\sqrt{3}}{2}\right)(m) + (0)(m) = p(m) + q(m)$$

$$\frac{u\sqrt{3}}{2} = p + q \quad \ldots \boxed{u\sqrt{3} = 2p + 2q}$$

Coeff. of Res.:

$$\frac{p-q}{\frac{u\sqrt{3}}{2} - 0} = -e$$

$$p - q = -\frac{eu\sqrt{3}}{2}$$

$$\boxed{2p - 2q = -eu\sqrt{3}}$$

Q5
(b)

$$2p + 2q = u\sqrt{3}$$
$$2p - 2q = -eu\sqrt{3}$$

$$4p = u\sqrt{3} - eu\sqrt{3}$$

$$p = \frac{u\sqrt{3}(1-e)}{4}$$

$$2p + 2q = u\sqrt{3}$$

$$\cancel{2}\left[\frac{u\sqrt{3}(1-e)}{\cancel{4}_{2}}\right] + 2q = u\sqrt{3} \qquad (\times 2)$$

$$u\sqrt{3}(1-e) + 4q = 2u\sqrt{3}$$

$$u\sqrt{3} - eu\sqrt{3} + 4q = 2u\sqrt{3}$$

$$4q = 2u\sqrt{3} - u\sqrt{3} + eu\sqrt{3}$$
$$4q = u\sqrt{3} + eu\sqrt{3}$$

$$q = \frac{u\sqrt{3}(1+e)}{4}$$

Speed of ①: $\sqrt{\left(\frac{u\sqrt{3}(1-e)}{4}\right)^2 + \left(\frac{u}{2}\right)^2}$

Speed of ②: $\sqrt{\left(\frac{u\sqrt{3}(1+e)}{4}\right)^2 + (0)^2}$

Speed of ① = Speed of ②

$$\frac{3u^2(1-2e+e^2)}{16} + \frac{u^2}{4} = \frac{3u^2(1+2e+e^2)}{16} + 0 \qquad (\times 16)$$

$$3\cancel{u^2}(1-2e+e^2) + 4\cancel{u^2} = 3\cancel{u^2}(1+2e+e^2)$$

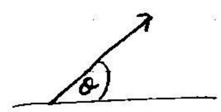
$$3 - 6e + \cancel{3e^2} + 4 = 3 + 6e + \cancel{3e^2}$$

$$3 + 4 - 3 = 12e.$$

$$4 = 12e$$

$$\frac{4}{12} = e \quad \ldots \quad \boxed{\frac{1}{3} = e}$$

(ii) Velocity of ① after collision:



$$\frac{u\sqrt{3}(1-e)}{4}\vec{i} + \frac{u}{2}\vec{j} \qquad \text{Let } e = \frac{1}{3}$$

$$\frac{u\sqrt{3}(1-\frac{1}{3})}{4}\vec{i} + \frac{u}{2}\vec{j}$$

$$\frac{u\sqrt{3}(\frac{2}{3})}{4}\vec{i} + \frac{u}{2}\vec{j} \Rightarrow \frac{2u\sqrt{3}}{12}\vec{i} + \frac{u}{2}\vec{j}$$

$$\tan\theta = \frac{\vec{j}}{\vec{i}} = \frac{\frac{u}{2}}{\frac{2u\sqrt{3}}{12}} = \frac{\cancel{u}}{2} \times \frac{\cancel{12}^{\,3}}{2\cancel{u}\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

So, $\tan\theta = \sqrt{3} \quad \ldots \quad \boxed{\theta = 60^\circ}$