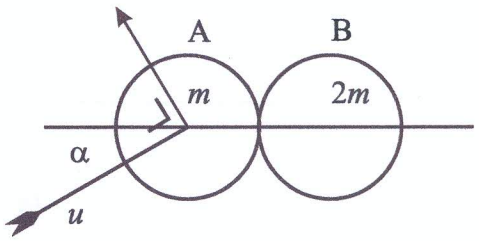
(b) A smooth sphere A, of mass m, moving with speed u, collides with a smooth sphere B, of mass $2m$, which is at rest. The direction of motion of A, before impact, makes an angle α with the line of centres at impact, where $0° \le \alpha < 90°$. As a result of the collision, the direction of A is deflected through an angle of 90°. The coefficient of restitution between the spheres is e.



(i) Show that $\tan\alpha = \sqrt{\dfrac{2e-1}{3}}$.

(ii) Find e, if the magnitude of the impulse exerted by A on B is $mu\cos\alpha$.

(i) PCM $\quad mu\cos\alpha + 2m(0) = mv_1 + 2mv_2$ — 5

NEL $\quad v_1 - v_2 = -e(u\cos\alpha - 0)$ — 5

$$v_1 = \frac{u\cos\alpha\{1-2e\}}{3}$$

$$v_2 = \frac{u\cos\alpha\{1+e\}}{3}$$ — 5

$$\tan\alpha = \frac{-v_1}{u\sin\alpha}$$

$$= \frac{-u\cos\alpha\{1-2e\}}{3u\sin\alpha}$$

$$\tan^2\alpha = \frac{2e-1}{3}$$ — 5

$$\tan\alpha = \sqrt{\frac{2e-1}{3}}$$

(ii) Impulse exerted by A on B $= |2m(0) - 2mv_2|$

$$mu\cos\alpha = \frac{2mu\cos\alpha\{1+e\}}{3}$$

$$3 = 2 + 2e$$

$$\Rightarrow \quad e = \tfrac{1}{2}$$ — 5

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