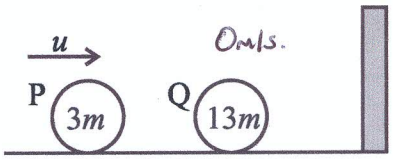
2003 HL

5. **(a)** A smooth sphere P, of mass $3m$, moving with speed u, collides directly with a smooth sphere Q, of mass $13m$, which is at rest. Sphere Q then collides with a vertical wall which is perpendicular to the direction of motion of the spheres. The coefficient of restitution for all of the collisions is e.

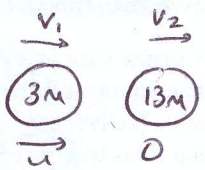


Find

(i) the speed, in terms of u and e, of each sphere after the first collision

(ii) the range of values of e for which there will be a second collision between the spheres.

1st collision P with Q



(i) PCM $\quad 3m(u)+13m(0) = 3mv_1+13mv_2$ ① ÷m $\quad 3 = 3v_1+13v_2$ — 5

NEL $\quad v_1 - v_2 = -e(u-0)$ — 5

$$v_1 = \frac{u}{16}(3-13e)$$

$$v_2 = \frac{3u}{16}(1+e)$$ — 5

① $3v_1+13v_2 = 3u$

13×② $13v_1-13v_2 = -13eu$

$16v_1 = 3u-13eu$

$v_1 = \frac{(3-13e)u}{16}$

sub into ②

$v_2 = \frac{(3-13e)u}{16} + eu$

$= \frac{3u-13eu+16eu}{16}$

$v_2 = \frac{3eu+3u}{16} = \frac{3u}{16}(e+1)$

(ii) There will be a second collision if :

$$ev_2 > -v_1$$

$$e\frac{3u}{16}(1+e) > -\frac{u}{16}(3-13e)$$ — 5

$$3e^2-10e+3 > 0$$

$$(3e-1)(e-3) > 0$$

$$\Rightarrow \quad 0 \le e < \tfrac{1}{3}$$ — 5

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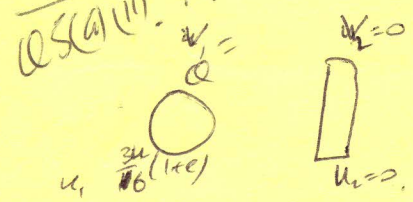
2003 HL Q5(a)(iii)



$v_1 - v_2 = -e(u_1 - u_2)$

$v_1 = -eu_1$

$-e\left(\frac{3u}{6}(1+e)\right) < v_1$