(b) A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle α with the line of centres at impact, where $0° \leq \alpha < 90°$.
The velocity of P is deflected through an angle ϑ by the collision, so that its velocity after impact makes an angle $\vartheta + \alpha$ with the line of centres at impact.
The coefficient of restitution between the spheres is $\frac{1}{4}$.

Show that $\tan\vartheta = \dfrac{5\tan\alpha}{3+8\tan^2\alpha}$.

PCM $\quad mu\cos\alpha + m(0) = mv_1 + mv_2$ $\quad$ 5

NEL $\quad v_1 - v_2 = -\frac{1}{4}(u\cos\alpha - 0)$ $\quad$ 5

$\Rightarrow \quad v_1 = \frac{3}{8}u\cos\alpha$ $\quad$ 5

$\tan(\vartheta+\alpha) = \dfrac{u\sin\alpha}{\frac{3}{8}u\cos\alpha}$ $\quad$ 5

$\dfrac{\tan\vartheta+\tan\alpha}{1-\tan\vartheta\tan\alpha} = \dfrac{8\tan\alpha}{3}$

$3\tan\vartheta + 3\tan\alpha = 8\tan\alpha - 8\tan\vartheta\tan^2\alpha$

$\Rightarrow \quad \tan\vartheta = \dfrac{5\tan\alpha}{3+8\tan^2\alpha}$ $\quad$ 5 $\quad$ 25