2001 HL

(b) Two identical smooth spheres, each of mass m and moving in the same direction collide directly. The coefficient of restitution between the spheres is e.
If u is the magnitude of the relative velocity between the spheres before impact, show that

(i) each sphere receives an impulse of magnitude $\frac{1}{2}mu(1+e)$

(ii) the loss in the total kinetic energy of the two spheres due to the impact is $\frac{1}{4}mu^2(1-e^2)$.

Relative to second sphere :

PCM $\quad mu + m(0) = mv_1 + mv_2$ — 5

NEL $\quad v_1 - v_2 = -e(u-0)$ — 5

$v_1 + v_2 = u$

$v_1 - v_2 = -eu$

$v_1 = \frac{1}{2}u(1-e)$

$v_2 = \frac{1}{2}u(1+e)$ — 5

$\left\{\text{Impulse} = |mv_2 - mu_2| = \frac{1}{2}mu(1+e)\right\}$

OR

$\left\{\begin{aligned}\text{Impulse} &= |mv_1 - mu_1| \\ &= \left|\tfrac{1}{2}mu(1-e) - mu\right| \\ &= \left|-\tfrac{1}{2}mu(1+e)\right| \\ &= \tfrac{1}{2}mu(1+e)\end{aligned}\right\}$ — 5

$\text{Loss in KE} = \frac{1}{2}m\{u_1^2 + u_2^2 - v_1^2 - v_2^2\}$

$= \frac{1}{2}m\left\{u^2 + 0 - \left\{\tfrac{1}{2}u(1-e)\right\}^2 - \left\{\tfrac{1}{2}u(1+e)\right\}^2\right\}$

$= \frac{1}{2}m\left\{u^2 - \tfrac{1}{4}u^2\{1-2e+e^2+1+2e+e^2\}\right\}$

$= \frac{1}{4}mu^2[1-e^2]$ — 5

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