1987

Collisions (Honours):

(5). Hawes (Text).

$\vec{v}_1 = x\vec{i} + 4\vec{j}$ $\vec{v}_2 = y\vec{i} - p\vec{j}$.

(4kg) (8kg) $e = 0.4$.

$\vec{u}_1 = -3\vec{i} + 4\vec{j}$ $\vec{u}_2 = -\frac{9}{2}\vec{i} + p\vec{j}$.

(Smoothness $\Rightarrow$ $\vec{j}$ cpt of velocities unchanged by collision)

(i) Find $\vec{v}_1$ and $\vec{v}_2$: (Find x and p)

NLR (in $\vec{i}$ dirn): $\Rightarrow$ $y - x = -(-4)(-\frac{9}{2} - 3.)$

$\Rightarrow$ $y - x = +(4)(\frac{15}{2})$.

$\Rightarrow$ $y - x = 3$. (1)

PCM: $\Rightarrow$ $4(3) + 8(-\frac{9}{2}) = 4x + 8y$

$\Rightarrow$ $12 - 36 = 4x + 8y$

$-24 = 4x + 8y$

$\Rightarrow$ $x + 2y = -6$ (2).

Solve (1) and (2): Add $\Rightarrow$ $3y = -3$.

$\boxed{y = -1.}$

$\therefore$ (2) $\Rightarrow$ $x + 2(-1) = -6$ $\Rightarrow$ $\boxed{x = -4}$

$\therefore$ $\vec{v}_1 = -4\vec{i} + 4\vec{j}$ and and $\vec{v}_2 = -1\vec{i} - p\vec{j}$

(Can't work out p yet.)

(ii) KE Loss: KE Before $= \frac{1}{2}4(3^2+4^2) + \frac{1}{2}8((-\frac{1}{2})^2 + (-p)^2)$.

$= 2(25) + 4(\frac{81}{4} + p^2)$

$= 50 + 81 + p^2 = 131 + p^2$ Joules

$\boxed{KE = \frac{1}{2}m|\vec{v}|^2}$

KE After $= \frac{1}{2}4((-4)^2 + (4)^2) + \frac{1}{2}8((-1)^2 + (-p)^2)$

$= 2(16+16) + 4(1 + p^2)$

$= 64 + 4 + p^2 = 68 + p^2$ Joules

$\therefore$ $\Delta KE = 131 + p^2 - (68 + p^2) = 63$ Joules qed.

(iii) If spheres move at right angles then $\vec{v}_1 \perp \vec{v}_2 \Leftrightarrow \vec{v}_1 \cdot \vec{v}_2 = 0$.

$\therefore$ $(-4\vec{i} + 4\vec{j}) \cdot (-1\vec{i} - p\vec{j}) = 0$.

$\Rightarrow (-4)(-1) + 4(-p)$

$\Rightarrow 4 - 4p = 0$.

$\Rightarrow$ $\boxed{p = 1}$

(Nice question!)