1986 (others)

Q5 (a)

(i) $v_1 = \frac{1}{2}(4u) = 2u$ find e:

NLR: $v_2 - v_1 = -e(u_2 - u_1)$

$\Rightarrow v_2 - 2u = -e(2u - 4u)$

$v_2 - 2u = -e(-2u)$

$v_2 - 2u = e\,2u$

$v_2 = (1+e)2u.$

PCM: $3m[4u] + 5m[2u] = 3m[v_1] + 5m[v_2]$

$\Rightarrow 12u + 10u = 6u + 5v_2$

$\Rightarrow 16u = 5v_2.$

$\rightarrow \frac{16u}{5} = v_2$

Equate ① and ② $\Rightarrow (1+e)2u = \frac{16u}{5}.$

$\Rightarrow (1+e) = \frac{8}{5}$

$\Rightarrow e = \frac{8}{5} - 1$

$\Rightarrow \boxed{e = \frac{3}{5}}$

(ii) v_1, v_2 any values:

NLR: $v_2 - v_1 = -e(2u - 4u)$

$\boxed{v_2 - v_1 = e\,2u.}$ ①

PCM: $3m(4u) + 5m(2u) = 3m(v_1) + 5m(v_2)$

$\Rightarrow \boxed{22u = 3v_1 + 5v_2}$ ②

Solve ① and ②: ① $\times 3 \Rightarrow -3v_1 + 3v_2 = e\,6u.$

② $\Rightarrow 3v_1 + 5v_2 = 22u$

$8v_2 = (6e + 22)u$

$\therefore v_2 = \left(\frac{22+6e}{8}\right)u$

As e is > 0 always $\Rightarrow v_2 > \frac{22}{8}u$

as $22 > 16 \Rightarrow v_2 > \frac{16}{8}u$

$\Rightarrow \boxed{v_2 > 2u}$ qed.