1982 (Hons)

Q4(a) $v_1 = ?\vec{i}$ $\quad v_2 = ?$

(10) (50)

$u_1 = 10\vec{i}$ $\quad u_2 = -5\vec{i}$.

PCM ⇒

$10(10) + 50(-5) = 10v_1 + 50v_2$

$\Rightarrow \quad -15 = v_1 + 5v_2 \quad (1)$

NLR ⇒

$v_2 - v_1 = -\frac{1}{2}[-5-10]$

$\Rightarrow \quad v_2 - v_1 = 7.5 \quad (2)$

(1)+(2) ⇒ $\quad 6v_2 = -7.5$

$\Rightarrow \quad \underline{v_2 = -1.25\vec{i}}$

$\Rightarrow \quad \underline{v_1 = -8.75\vec{i}}$.

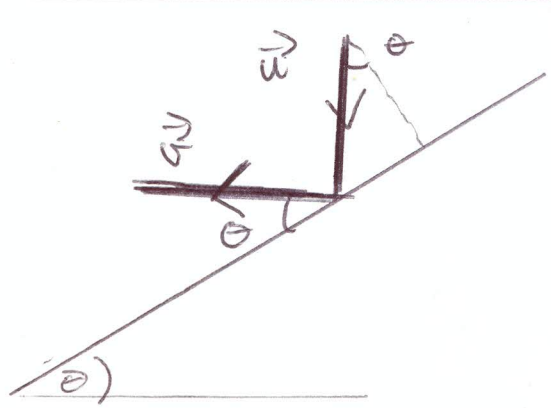
Impulse on 10Kg =

$10(-8.75\vec{i}) - 10(10\vec{i})$

$= -87.5 - 100\vec{i}$

$= -187.5\vec{i}$ Ns.

(b)



$\vec{u} = u\sin\theta\vec{i} - u\cos\theta\vec{j}$.

$\vec{v} = u\sin\theta\vec{i} + v\vec{j}$

(Smoothness ⇒ $\vec{i}$ cpt unchanged here)

NLR $\quad e = \frac{2}{3}$

$\Rightarrow y = -\frac{2}{3}(-u\cos\theta - 0)$

$\Rightarrow y = \frac{2}{3}u\cos\theta$.

Calculate ΔKE

KE before: [$\vec{j}$ dir^n only]

$\frac{1}{2}m(u\cos\theta)^2 = m\frac{1}{2}u^2\cos^2\theta$

KE After: [$\vec{j}$ dir^n only]

$\frac{1}{2}m\left[\frac{2}{3}u\cos\theta\right]^2 = m\frac{2}{9}u^2\cos^2\theta$

$\Delta KE = m\frac{1}{2}u^2\cos^2\theta - m\frac{2}{9}u^2\cos^2\theta$

$= \frac{1}{2}mu^2\cos^2\theta\left[1 - \frac{4}{9}\right]$

$= \frac{1}{2}mu^2\cos^2\theta\left[\frac{5}{9}\right]$

Fractional KE = $\frac{\Delta KE}{\text{original KE}}$

original KE [Both cpts]

$= \frac{1}{2}m\left[(u\sin\theta)^2 + (u\cos\theta)^2\right]$

$= \frac{1}{2}m\left[u^2(\sin^2\theta + \cos^2\theta)\right]$

$= \frac{1}{2}mu^2$

$\therefore$ Fractional KE loss = $\frac{\frac{1}{2}mu^2\cos^2\theta\left(\frac{5}{9}\right)}{\frac{1}{2}mu^2}$

$= \cos^2\theta\left(\frac{5}{9}\right)$.

But Geometry [Special] of $\vec{v}$.

$\Rightarrow \tan\theta = \frac{v}{u\sin\theta}$

$\Rightarrow \tan\theta = \frac{\frac{2}{3}u\cos\theta}{u\sin\theta}$

$\Rightarrow \tan\theta = \frac{2}{3}\frac{1}{\tan\theta} \Rightarrow \boxed{\tan^2\theta = \frac{2}{3}}$

Also $\cos^2\theta = [1+\tan^2\theta]^{-1} \Rightarrow \cos^2\theta = \left(\frac{5}{3}\right)^{-1} = \frac{3}{5}$

$\Rightarrow$ Frac KE = $\frac{3}{5}\left(\frac{5}{9}\right) = \frac{1}{3}$!!!!