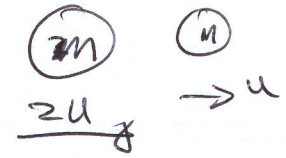
1981 Q4 :

(i)



NLR: $v_2 - v_1 = -e(u_2 - u_1)$

$v_2 - v_1 = -e(u - 2u)$

$v_2 - v_1 = eu$ ①

PCM: $m(2u) + m(u) = mv_1 + mv_2$

$3u = v_1 + v_2$ ②

Add: $2v_2 = eu + 3u$

$\Rightarrow v_2 = \frac{u(e+3)}{2}$

$\therefore$ ② $\Rightarrow 3u = v_1 + \frac{u(e+3)}{2}$

$\Rightarrow 3u - \frac{u(e+3)}{2} = v_1$

$\Rightarrow \frac{6u - ue - 3u}{2} = v_1 \Rightarrow v_1 = \frac{3u - eu}{2}$

$\Rightarrow v_1 = \frac{u(3-e)}{2}$.

ΔKE : KE before $= \frac{1}{2}m(u)^2 + \frac{1}{2}m(2u)^2 = \frac{5mu^2}{2}$.

KE after $= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$= \frac{1}{2}m\left(\frac{ue+3u}{2}\right)^2 + \frac{1}{2}m\left(\frac{3u-ue}{2}\right)^2$

$= \frac{1}{8}m\left(u^2e^2 + 6ue + 9u^2\right) + \frac{1}{8}m\left(9u^2 - 6ue + u^2e^2\right)$

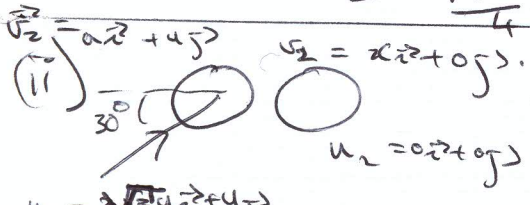
$= \frac{1}{8}mu^2\left[e^2 + 6e + 9 + 9 - 6e + e^2\right]$

$= \frac{1}{8}mu^2\left[2e^2 + 18\right] = \frac{1}{4}mu^2\left[e^2 + 9\right]$

$\therefore \Delta KE = \frac{5}{2}mu^2 - \frac{1}{4}mu^2\left[e^2 + 9\right]$

$= \frac{mu^2}{4}\left[10 - e^2 - 9\right] = \frac{mu^2}{4}\left[1 - e^2\right]$ qed

(ii)



PCM: $\Rightarrow m(\sqrt{3}u) + 0 = m(a) + m(x)$

$\sqrt{3}u = a + x$ ①

NLR: $\Rightarrow v_2 - v_1 = -e(u_2 - u_1)$

$\Rightarrow x - a = -e(0 - \sqrt{3}u)$ ②

Add: $2x = \sqrt{3}u + e\sqrt{3}u \Rightarrow x = \sqrt{3}u\left(\frac{1+e}{2}\right)$

$\therefore a + \sqrt{3}u\left(\frac{1+e}{2}\right) = \sqrt{3}u$

$a = 2\frac{\sqrt{3}u}{2} - \frac{\sqrt{3}u}{2} - \frac{\sqrt{3}eu}{2} \Rightarrow a = \frac{\sqrt{3}u}{2} - \frac{\sqrt{3}eu}{2} - \frac{\sqrt{3}u}{2}(1-e)$

$\therefore$ KE before $\frac{1}{2}m(\sqrt{3}u)^2 = \frac{3mu^2}{2}$.

KE after: $\frac{1}{2}ma^2 + \frac{1}{2}mx^2 = \frac{1}{2}m\left(\sqrt{3}u\left(\frac{1+e}{2}\right)\right)^2 + \frac{1}{2}m\left(\sqrt{3}u\left(\frac{1-e}{2}\right)\right)^2$

$= \frac{3mu^2}{8}\left(1 + 2e + e^2 + 1 - 2e + e^2\right)$

$= \frac{3mu^2}{4}\left(1 + e^2\right)$

ΔKE BOARD $= \frac{3}{2}mu^2 - \frac{3mu^2}{4}(1+e^2) = \frac{3}{4}mu^2(1-e^2)$.