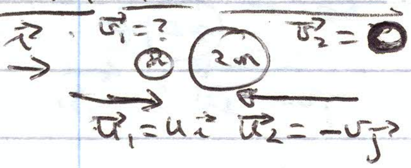
1979. Q7 Collisions



PCM $\Rightarrow$ $mu + 2m(-v) = mv_1 + 2m(0)$

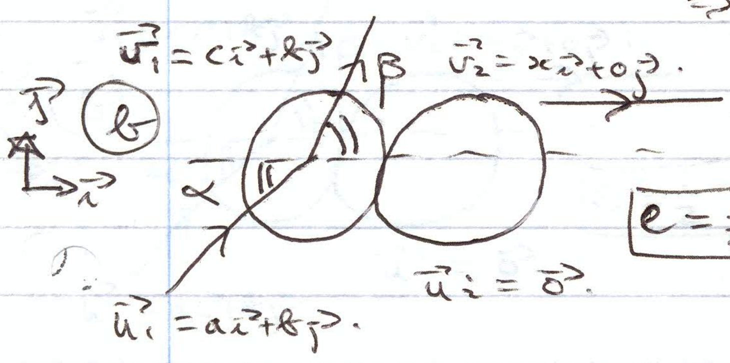
$\Rightarrow$ $\boxed{u - 2v = v_1}$ (i)

NLR $\Rightarrow$ $v_2 - v_1 = -e(u_2 - u_1)$

$0 - v_1 = -e(-v - u)$

(i) $\Rightarrow$ $-(u - 2v) = -e(-v - u)$

$\Rightarrow$ $e = \dfrac{2v - u}{u + v}$ qed



Smoothness $\Rightarrow$ $\vec{j}$ cpt of speeds unchanged

$\boxed{e = \frac{1}{2}}$ (-1) $\boxed{\frac{b}{a} = \operatorname{Tan}\alpha}$, $\boxed{\frac{b}{c} = \operatorname{Tan}\beta}$ (0)

"geometry" Equations

PCM applied to $\vec{i}$ dir^n $\Rightarrow$ $ma + m0 = mc + mx$

$\Rightarrow$ $a = c + x$. — (1)

NLR applied to $\vec{j}$ dir^n $\Rightarrow$ $v_2 - v_1 = -e(u_2 - u_1)$

$\Rightarrow$ $x - c = -\frac{1}{2}(0 - a)$

$\Rightarrow$ $x - c = \frac{a}{2}$ (2)

Now review what you were asked to do!

Prove $\tan\beta = 4\tan\alpha$.

"geometry" equations (-1) and (0) $\Rightarrow$ $b = a\operatorname{Tan}\alpha$

and $b = c\operatorname{Tan}\beta$

$\therefore$ $\underline{a\operatorname{Tan}\alpha = c\operatorname{Tan}\beta}$. (4)

So if can get c in terms of a we could finish.

So eliminate x from (1) and (2).

Subtract: $c + c = a - \frac{a}{2}$ $\Rightarrow$ $2c = \frac{a}{2}$ $\Rightarrow$ $\boxed{c = \frac{a}{4}}$

So (4) $\Rightarrow$ $a\operatorname{Tan}\alpha = \frac{a}{4}\operatorname{Tan}\beta$

$\Rightarrow$ $\underline{4\operatorname{Tan}\alpha = \operatorname{Tan}\beta}$ qed.