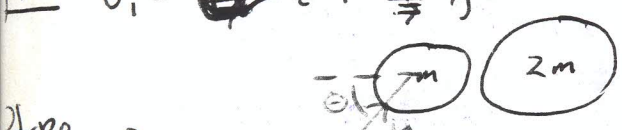
1978 Q3 (collisions)

$a\vec{i}+b\vec{j}$ is $\perp$ to $c\vec{i}+d\vec{j}$ $\Leftrightarrow$ $(a\vec{i}+b\vec{j})\cdot(c\vec{i}+d\vec{j})=0$ This is what!

$\Leftrightarrow$ $\boxed{ac+bd=0.}$

After $\vec{v}_1 = v\vec{i}+\frac{\sqrt{40}u}{7}\vec{j}$ $\quad \vec{v}_2 = x\vec{i}+0\vec{j}$



$\vec{v}_1 = -v\vec{i}+0\vec{j}$

$\boxed{e=\frac{3}{4}}$

$\vec{j}$ cpts unchanged because of smoothness

Before $\vec{u}_1 = u\cos\theta\,\vec{i}+u\sin\theta\,\vec{j}$

$\vec{u}_1 = \frac{3u}{7}\vec{i}+\frac{\sqrt{40}u}{7}\vec{j}$.

ALSO: $\vec{v}_1 \perp \vec{u}_1 \Leftrightarrow \vec{v}_1\cdot\vec{u}_1 = 0.$

$\Rightarrow (a\vec{i}+\frac{\sqrt{40}u}{7}\vec{j})\cdot(\frac{3u}{7}\vec{i}+\frac{\sqrt{40}u}{7}\vec{j}) = 0$

$\Rightarrow a\frac{3u}{7} = -\frac{40}{49}u$

$\Rightarrow \boxed{a=-\frac{40}{21}u}$

PCM ($\vec{i}$ dir^n.) $\Rightarrow m(\frac{3u}{7})+2m(-v) = m(-\frac{40u}{21})+2m(x)$

$\Rightarrow \frac{3u}{7}-2v = -\frac{40}{21}u+2x$

$\times 21 \Rightarrow 9u-42v = -40u+42x$

$\Rightarrow \boxed{42x+42v=+49u}$ (1)

NLR ($\vec{i}$ dir^n) $\Rightarrow v_2-v_1 = -e(u_2-u_1)$

$\Rightarrow x-(-\frac{40u}{21}) = -\frac{3}{4}(-v-\frac{3u}{7})$

$\Rightarrow x+\frac{40u}{21} = \frac{3v}{4}+\frac{9u}{28}$

$\times 84 \Rightarrow 84x+160u = 63v+27u.$

$\Rightarrow \boxed{84x-63v=-133u}$ (2)

Asked to show $v=\frac{11}{7}u$ so eliminate x from (1) and (2)

$2\times$(1) $-$ (2) $\Rightarrow$

$84x+84v = 98u$

$84x-63v = -133u$

$147v = 231u$

$\Rightarrow v=\frac{231}{147}u \Rightarrow \boxed{v=\frac{11}{7}u}$ q.e.d.

Dir^n of sphere moves: After collision $\vec{v}_1 = -\frac{40}{21}u\,\vec{i}+\frac{\sqrt{40}u}{7}\vec{j}$

$\Rightarrow \theta = \frac{\frac{\sqrt{40}}{7}}{\frac{40}{21}} = \frac{\sqrt{40}}{7}\times\frac{21}{40} = \frac{3}{\sqrt{40}}$

$\therefore$ Dir^n is NW of N when $\tan\theta = \frac{3}{\sqrt{40}}$.