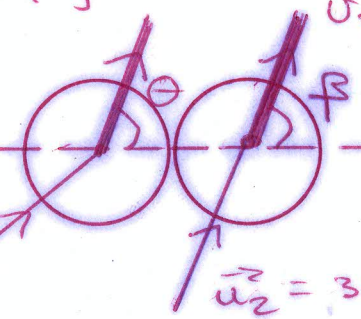
HONS COLLISIONS (1977)

$\vec{v}_1 = a\vec{i} + 2\vec{j}$. $\quad \vec{v}_2 = x\vec{i} + 4\vec{j}$.



$M = 3kg$ $\quad m = 6kg$ $\quad e = \frac{3}{4}$

$\vec{u}_1 = 5\vec{i} + 2\vec{j}$. $\quad \vec{u}_2 = 3\vec{i} + 4\vec{j}$.

Trig:

$\tan A = \frac{2}{5}$

$\Rightarrow \cos A = \frac{5}{\sqrt{29}}$ $\quad \sin A = \frac{2}{\sqrt{29}}$.

$\Rightarrow \vec{u}_1 = \sqrt{29}(\cos A)\vec{i} + \sqrt{29}(\sin A)\vec{j}$

$= 5\vec{i} + 2\vec{j}$

$\tan B = \frac{4}{3}$

$\Rightarrow \cos B = \frac{3}{5}$ $\quad \sin B = \frac{4}{5}$.

$\vec{u}_2 = 5\cos B\,\vec{i} + 5\sin B\,\vec{j}$.

$\vec{u}_2 = 3\vec{i} + 4\vec{j}$

Smoothness $\Rightarrow$ $\vec{j}$ cpts of velocities remain unchanged

PCM ($\vec{i}$ dirn) $\Rightarrow$ $3(5) + 6(3) = 3a + 6x$ ①

$\Rightarrow 3a + 6x = 33$

NLR ($\vec{i}$ dirn) $\Rightarrow$ $a - x = -\frac{3}{4}(5-3)$

$\Rightarrow 4a - 4x = -6$.

$\Rightarrow 2a - 2x = -3$ ②

Solve ① and ②: $\quad 3a + 6x = 33$, $\quad 6a - 6x = -9$ $\Rightarrow 9a = 24$.

$\Rightarrow a = \frac{24}{9} = \frac{8}{3}$.

Sub $\frac{8}{3}$ $\Rightarrow$ $3\left(\frac{8}{3}\right) + 6x = 33 \Rightarrow 6x = 25 \Rightarrow x = \frac{25}{6}$

$\therefore$ $\boxed{\vec{v}_1 = \frac{8}{3}\vec{i} + 2\vec{j}}$ $\quad \boxed{\vec{v}_2 = \frac{25}{6}\vec{i} + 4\vec{j}}$.

Speeds: $v_1 = \sqrt{\left(\frac{8}{3}\right)^2 + 2^2} = \sqrt{\frac{64}{9} + 4}$

$= \sqrt{\frac{64}{9} + \frac{36}{9}}$

$= \sqrt{\frac{100}{9}}$

$v_1 = \frac{10}{3}$. ms^{-1}

$v_2 = \sqrt{\left(\frac{25}{6}\right)^2 + 4^2}$

$= \sqrt{\frac{625}{36} + \frac{576}{36}} = \sqrt{\frac{1201}{36}} = 5.776\,ms^{-1}$

$\boxed{v_2 = 5.776\,ms^{-1}}$

dirn: $\vec{v}_1$: $\tan\theta = \frac{4}{2} = 2 \Rightarrow \theta = 63.43°$.

Dirn E 53.13° N .

dirn $\vec{v}_2$: $\tan\beta = \frac{4}{\frac{25}{6}} = \frac{24}{25} = .96$

$\Rightarrow \beta = 43.83°$.

$\Rightarrow$ dirn is E 43.83° N .

KE loss:

$KE_{Before} = \frac{1}{2}\left[3u_1^2 + 6u_2^2\right] = \frac{1}{2}\left[3(\sqrt{29})^2 + 6(5)^2\right] = 43.5 + 125$

$= 168.5$ Joules

$KE_{After} = \frac{1}{2}\left[3v_1^2 + 6v_2^2\right] = \frac{1}{2}\left[3(5.776)^2 + 3(3.334)^2\right]$

$= 50.04 + 33.36$

$= 83.4$

$KE_{Loss} = (168.5 - 83.4)$ Joules $= 1\frac{3}{4}$ J .

$= 85.1$ Joules