1983, Q2 ctd

$\Rightarrow u_1 u_2 = x^2 \cos^2\alpha - \frac{v^2}{2}$

$\Rightarrow u_1 u_2 = x^2(1-\sin^2\alpha) - \frac{v^2}{2}$ (Because $\cos^2\alpha + \sin^2\alpha = 1$)

$\Rightarrow u_1 u_2 = x^2\left(1-\left(\frac{v}{x\sqrt{2}}\right)^2\right) - \frac{v^2}{2}$ (Because of Eqn (H))

$\Rightarrow u_1 u_2 = x^2\left(1-\frac{v^2}{x^2 2}\right) - \frac{v^2}{2}$

$\Rightarrow u_1 u_2 = x^2 - \frac{v^2}{2} - \frac{v^2}{2}$

$\Rightarrow u_1 u_2 = x^2 - v^2$ qed (!)

Expression for Time for return journey: Time $= \frac{d}{u_1} + \frac{d}{u_2}$ Speed u_1 and u_2

We could do either of the three methods that follow

WAY ONE: (SLOG)

Find u_1, u_2 by solving

$u_1 - u_2 = v\sqrt{2}$ (A)

$u_1 u_2 = x^2 - v^2$ (B)

Non linear simultaneous equations in (A) and (B)

(A) $\Rightarrow u_1 = v\sqrt{2} + u_2$

(B) $\Rightarrow$

$(v\sqrt{2}+u_2)u_2 = x^2 - v^2$

$\Rightarrow v\sqrt{2}u_2 + u_2^2 = x^2 - v^2$

$\Rightarrow u_2^2 + v\sqrt{2}u_2 - (x^2-v^2) = 0$

Quadratic in u_2 with

$a = 1 \quad b = v\sqrt{2} \quad c = -x^2+v^2$

$u_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$u_2 = \frac{-v\sqrt{2} \pm \sqrt{(v\sqrt{2})^2 - 4(-x^2+v^2)}}{2}$

$u_2 = \frac{-v\sqrt{2} \pm \sqrt{2v^2 - 4v^2 + 4x^2}}{2}$

$u_2 = \frac{-v\sqrt{2} \pm \sqrt{4x^2-2v^2}}{2}$

As u_2 is supposed to be +ve take the + option

$u_2 = \frac{-v\sqrt{2} + \sqrt{4x^2-2v^2}}{2}$

$\therefore u_1 = v\sqrt{2} + \frac{-v\sqrt{2}}{2} + \frac{\sqrt{4x^2-2v^2}}{2}$

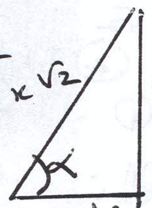
$\Rightarrow u_1 = \frac{v\sqrt{2}}{2} + \frac{\sqrt{4x^2-2v^2}}{2}$

And $T = \frac{d}{u_1} + \frac{d}{u_2}$

with these subbed in

WAY TWO: (TRIG)

If $\sin\alpha = \frac{v}{x\sqrt{2}}$



$\cos\alpha = \frac{adj}{x\sqrt{2}}$

$v^2 + adj^2 = (x\sqrt{2})^2$

$\Rightarrow adj^2 = 2x^2 - v^2$

$adj = \sqrt{2x^2 - v^2}$

$\Rightarrow \cos\alpha = \frac{\sqrt{2x^2-v^2}}{x\sqrt{2}}$

$\Rightarrow$

$u_1 = x\cos\alpha + \frac{v}{\sqrt{2}}$

$u_1 = x\frac{\sqrt{2x^2-v^2}}{x\sqrt{2}} + \frac{v}{\sqrt{2}}$

$u_1 = \frac{\sqrt{2x^2-v^2}}{\sqrt{2}} + \frac{v}{\sqrt{2}}$

(as above)

and

$u_2 = +x\cos\alpha - \frac{v}{\sqrt{2}}$

$= x\frac{\sqrt{2v^2-x^2}}{x\sqrt{2}} - \frac{v}{\sqrt{2}}$

$u_2 = -\frac{v}{\sqrt{2}} + \frac{\sqrt{2v^2-x^2}}{\sqrt{2}}$

(as above)

and again $T = \frac{d}{u_1} + \frac{d}{u_2}$ with these values in!

WAY THREE: (GENIUS!)

$T = \frac{d}{u_1} + \frac{d}{u_2}$

$\Rightarrow T = d\left(\frac{1}{u_1} + \frac{1}{u_2}\right)$

$\Rightarrow T = d\left(\frac{u_2+u_1}{u_1 u_2}\right)$

$\Rightarrow T^2 = d^2 \frac{(u_1+u_2)^2}{(u_1 u_2)^2}$

But $(u_1+u_2)^2 = (u_1-u_2)^2 + 4u_1u_2$

$\Rightarrow (u_1+u_2)^2 = (v\sqrt{2})^2 + 4(x^2-v^2)$

$\Rightarrow (u_1+u_2)^2 = 2v^2 + 4x^2 - 4v^2$

$= 4x^2 - 2v^2$

$\Rightarrow T^2 = \frac{d^2(4x^2-2v^2)}{(x^2-v^2)^2}$

$\Rightarrow T = \frac{d\sqrt{4x^2-2v^2}}{x^2-v^2}$

...