1985 Q10

$3y^2(x-1) = 1+y^3$

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$3y^2(x-1) = 1+y^3$ ⇒

$$\int \frac{3y^2\,dy}{1+y^3} = \int \frac{dx}{x-1} + A$$

$$\Rightarrow \ln(1+y^3) = \ln(x-1) + A.$$

// $x=0$ when $y=0$ ⇒ $\ln(1+0^3) = \ln|0-1| + A$

$0 = 0 + A$

$0 = A$ //

NB [Ln only defined for +ves.]

$$\Rightarrow \ln(1+y^3) = \ln(x-1)$$

$$\Rightarrow 1+y^3 = x-1.$$

$$\Rightarrow y = \sqrt[3]{x-2}.$$

6(i) Positions:

O	P
$t=0$	$t=T$
$x=0$	$x=X$
$v=u$	$v=\frac{u}{3}$

Forces: $-mkv^2$ ←

Average Speed $= \frac{X-0}{T-0} = \frac{X}{T}$.

NII: $ma = -mkv^2$

$$\frac{d^2x}{dt^2} = -kv^2$$

Link t and v.

$$\frac{dv}{dt} = -kv^2$$

$$\int_u^{\frac{u}{3}} \frac{dv}{v^2} = \int_0^T -k\,dt$$

$$-\frac{1}{v}\Big|_u^{\frac{u}{3}} = -kt\Big|_0^T$$

$$\frac{1}{v}\Big|_u^{\frac{u}{3}} = kt\Big|_0^T$$

$$\frac{3}{u} - \frac{1}{u} = kT$$

$$\frac{2}{u} = kT$$

$$\frac{1}{k}\left(\frac{2}{u}\right) = T$$

Link x and v

$$v\frac{dv}{dx} = -kv^2. \quad (*)$$

$$\int_u^{\frac{u}{3}} \frac{dv}{v} = \int_0^X -k\,dx$$

$$\Rightarrow \ln v\Big|_u^{\frac{u}{3}} = -kx\Big|_0^X.$$

$$\Rightarrow \ln\tfrac{u}{3} - \ln u = -kX + 0.$$

$$\Rightarrow \ln\left[\tfrac{u}{3}/u\right] = -kX$$

$$\Rightarrow \ln\left(\tfrac{1}{3}\right) = -kX$$

$$\Rightarrow \ln 1 - \ln 3 = -kX$$

$$\Rightarrow -\ln 3 = -kX$$

$$\frac{1}{k}(\ln 3) = X$$

Average speed $= \frac{X}{T} = \frac{\frac{1}{k}\ln 3}{\frac{1}{k}\frac{2}{u}} = \frac{u}{2}\ln 3$ qed.

6(ii). Speed at midpoint.

O	
$x=0$	$x=\frac{X}{2}$
$t=0$	
$v=u$	$v=V$?

Link speed and distance:

$(*)$ ⇒

$$\int_u^V \frac{dv}{v} = \int_0^{\frac{X}{2}} -k\,dx$$

$$\Rightarrow \ln v\Big|_u^V = -kx\Big|_0^{\frac{X}{2}}$$

$$\Rightarrow \ln V - \ln u = -k\left[\tfrac{1}{k}\left(\tfrac{\ln 3}{2}\right)\right]$$

Tidy

$\ln\frac{V}{u} = -\frac{1}{2}\ln(3)$

$\ln\frac{V}{u} = \ln 3^{-\frac{1}{2}}$

$\Rightarrow \frac{V}{u} = (3)^{-\frac{1}{2}}$

$\Rightarrow \frac{V}{u} = \frac{1}{\sqrt{3}} \Rightarrow V = \frac{u}{\sqrt{3}}$