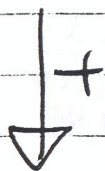
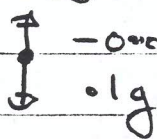


1974 Q10 : using Bands of integration.

SIGNS



Forces



START

$$\begin{aligned} x &= 0 \\ t &= 0 \\ v &= 0 \end{aligned}$$

typical
Point q

$$\begin{aligned} x &= X \\ t &= T \\ v &= V \end{aligned}$$

NI

$$F = ma.$$

$$\Rightarrow 0.1g - 0.002v = 0.1(\text{accel})$$

$$\Rightarrow 9.8 - 2v = 10(\text{accel})$$

$$\Rightarrow 49 - v = 5 \frac{dv}{dt} \quad \text{OR} \quad 49 - v = 5 \left(\frac{dv}{dx} \frac{dx}{dt} \right)$$

Find v as $t \rightarrow \infty$: Use Version (1) :

$$\Rightarrow \int_0^T \frac{dt}{5} = \int_0^V \frac{dv}{49-v}$$

$$\Rightarrow \frac{t}{5} \Big|_0^T = -\ln(49-v) \Big|_0^V$$

$$\Rightarrow \Rightarrow \frac{T}{5} = -\ln(49-V) + \ln(49-0)$$

$$\Rightarrow \frac{T}{5} = -\ln(49-V) + \ln 49$$

$$\Rightarrow -\frac{T}{5} = \ln(49-V) - \ln 49$$

$$\Rightarrow -\frac{T}{5} = \ln \left(\frac{49-V}{49} \right)$$

$$\Rightarrow e^{-\frac{T}{5}} = \frac{49-V}{49}$$

speed V at any time $\Rightarrow 49e^{-\frac{T}{5}} = 49-V$

$$\Rightarrow 49 - 49e^{-\frac{T}{5}} = V$$

As $T \rightarrow \infty \Rightarrow V \rightarrow 49 - 49e^{-\frac{T}{5}}$
 $(e^{-\frac{T}{5}} \rightarrow 0) \rightarrow 49 - 0$

$$V \rightarrow 49 \text{ ms}^{-1}$$

For later: $\int \frac{dv}{49-v}$: Let $u = 49-v \Rightarrow du = -dv$
 $\Rightarrow \int \frac{dv}{49-v} = \int \frac{-du}{u} = -\ln u$

Aside

$$\int \frac{v}{49-v} dv$$

$$\text{Let } u = 49-v$$

$$\Rightarrow v = 49-u$$

$$\text{and } dv = -du$$

$$\therefore \int \frac{v}{49-v} dv$$

$$= \int \frac{(49-u)(-du)}{u}$$

$$= \int \left(\frac{49-u}{u} \right) du$$

$$= \int \left(1 - \frac{49}{u} \right) du$$

$$= u - 49 \ln u$$

$$= (49-v) - 49 \ln(49-v)$$

Find X gone time T.

Either: Integrate again:

$$V = 49 - 49e^{-\frac{t}{5}}$$

$$\frac{dx}{dt} = 49 - 49e^{-\frac{t}{5}}$$

$$\Rightarrow \int_0^X dx = \int_0^T (49 - 49e^{-\frac{t}{5}}) dt$$

$$x \Big|_0^X = (49t + 49(5)e^{-\frac{t}{5}}) \Big|_0^T$$

$$X = (49T + 245e^{-\frac{T}{5}}) - (49(0) + 245e^0)$$

$$X = 49T + 245e^{-\frac{T}{5}} - 245$$

$$X = 49T + 245(e^{-\frac{T}{5}} - 1)$$

OR TRY

Version (2) $\Rightarrow 49-v = 5 \frac{dv}{dx}$

and $\int \frac{1}{5} dx = \int \frac{v}{49-v} dv$

$$\frac{x}{5} \Big|_0^X = \left[(49-v) - 49 \ln(49-v) \right]_0^V$$

Evaluate and sub for V in terms of T [messy!]