EXAM PAPERS: 1976 – ~~1983~~ – 1982

LEAVING CERTIFICATE EXAMINATION, 1976

APPLIED MATHEMATICS – HIGHER LEVEL

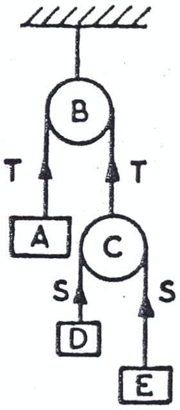
FRIDAY, 25 JUNE – MORNING, 9.30 to 12

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. Show that, if a particle is moving in a straight line with constant acceleration k and initial speed u, the distance travelled in time t is given by $s = ut + \frac{1}{2}kt^2$. Two points a and b are a distance l apart. A particle starts from a and moves towards b in a straight line with initial velocity u and constant acceleration k. A second particle starts at the same time from b and moves towards a with initial velocity $2u$ and constant deceleration k. Find the time in terms of u, l at which the particles collide, and the condition satisfied by u, k, l if this occurs before the second particle returns to b.

2. A particle is projected upwards with a speed of 35 m/s from a point 0 on a plane inclined at 45° to the horizontal. The plane of projection meets the inclined plane in a line of greatest slope and the angle of projection, measured to the inclined plane, is ϕ. Write down the velocity of the particle and its displacement from 0, in terms of $\vec{i}$ and $\vec{j}$, after time t seconds. If the particle is moving horizontally when it strikes the plane at q prove that $\cot\phi = 3$ and calculate $|oq|$.

3. The diagram shows a light inelastic string, passing over a fixed pulley B, connecting a particle A of mass 3M to a light movable pulley C. Over this pulley passes a second light inelastic string to the ends of which are attached particles D, E of masses 2M, M respectively. Show in separate diagrams the forces acting on A, D and E. Write down the three equations of motion involving the tensions T, S in the strings, the acceleration of A and the common acceleration of D, E *relative* to C. Show that T = 2S = 48Mg/17.



4. A light smooth ring of mass M is threaded on a smooth fixed vertical wire and is connected by a light inelastic string, passing over a fixed smooth peg at a distance l from the wire, to a particle of mass 2M hanging freely. The system is released from rest when the string is horizontal. Explain why the conservation of energy can be applied to the system. If the ring descends a distance of x while the particle rises through a distance y show that

$$x^2 = y^2 + 2ly \text{ and } (l + y)\dot{y} = x\dot{x}$$

where $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$ are the speeds of the ring and particle respectively. Find $\dot{x}$ when (i) $x = l$ and (ii) when $x = {}^{4l}/_{3}$.

5. State the laws governing the oblique collision of elastic spheres.

A sphere of mass M moving with speed u collides obliquely with a second smooth sphere at rest. The direction of motion of the moving sphere is inclined at 45° to the line of centres at impact, and the coefficient of restitution is $\frac{1}{2}$. After impact the directions of motion of the spheres are at right angles.

Find the mass of the second sphere in terms of M, and the velocities of the two spheres after impact in terms of u. Hence show that one quarter of the kinetic energy is lost.