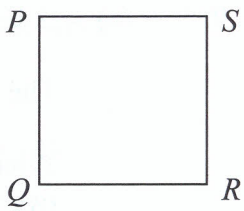
2011

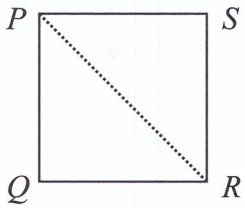
8. (b) A square lamina $PQRS$, of side 60 cm and mass m, can turn freely about a horizontal axis through P perpendicular to the plane of the lamina.
The lamina is released from rest when PS is horizontal.



(i) Find the angular velocity of the lamina when PR is vertical.

A mass m is attached to the lamina at R. The compound pendulum is set in motion.

(ii) Find the period of small oscillations of the compound pendulum and hence, or otherwise, find the length of the equivalent simple pendulum.



(i) Gain in KE = Loss in PE

$$\tfrac{1}{2}I\omega^2 = mgh$$

$$\tfrac{1}{2}\left\{\tfrac{4}{3}m(0.3)^2 + \tfrac{4}{3}m(0.3)^2\right\}\omega^2 = mg\left(0.3\sqrt{2} - 0.3\right)$$

$$\omega^2 = \frac{g\left(\sqrt{2}-1\right)}{0.4} = 10.1482$$

$$\omega = 3.19 \text{ rad s}^{-1}$$

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(ii)

$$I = \tfrac{8}{3}m(0.3)^2 + m\left(0.6\sqrt{2}\right)^2$$

$$= 0.96m$$

$$Mgh = mg\left(0.3\sqrt{2}\right) + mg\left(0.6\sqrt{2}\right)$$

$$= 0.9\sqrt{2}\,mg$$

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$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

$$= 2\pi\sqrt{\frac{0.96m}{0.9\sqrt{2}\,mg}}$$

$$= 1.74 \text{ s}$$

5

$$2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.96}{0.9\sqrt{2}\,g}}$$

$$\Rightarrow L = \frac{0.96}{0.9\sqrt{2}} = 0.75 \text{ m}$$

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