2011

8. (a) Prove that the moment of inertia of a uniform square lamina of mass m and side 2ℓ about an axis through its centre parallel to one of its sides is $\frac{1}{3}m\ell^2$.

Let M = mass per unit area

mass of element = $M\{2\ell\, dx\}$

moment of inertia of the element = $M\{2\ell\, dx\}x^2$ — 5

moment of inertia of the lamina = $2\ell M \int_{-\ell}^{\ell} x^2\, dx$ — 5

$= 2\ell M\left[\frac{x^3}{3}\right]_{-\ell}^{\ell}$ — 5

$= 4M\frac{\ell^4}{3}$

$= \frac{1}{3}m\ell^2$ — 5 — 20