2011

**6.** **(a)** The distance, $x$, of a particle from a fixed point, $O$, is given by

$$x = a\sin(\omega t + \varepsilon)$$

where $a, \omega$ and $\varepsilon$ are positive constants.

**(i)** Show that the motion of the particle is simple harmonic.

A particle moving with simple harmonic motion starts from a point 1 m from the centre of the motion with a speed of 9.6 $\text{m s}^{-1}$ and an acceleration of 16 $\text{m s}^{-2}$.

**(ii)** Calculate $a, \omega$ and $\varepsilon$.

(i)

$$x = a\sin(\omega t + \varepsilon)$$

$$\dot{x} = a\omega\cos(\omega t + \varepsilon)$$

$$\ddot{x} = -a\omega^2\sin(\omega t + \varepsilon)$$
$$= -\omega^2 x$$

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(ii)

$$\ddot{x} = \omega^2 x$$
$$16 = \omega^2(1)$$
$$\Rightarrow \omega = 4 \text{ rad s}^{-1}$$

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$$\dot{x} = a\omega\cos(\omega t + \varepsilon)$$
$$9.6 = a(4)\cos\varepsilon$$
$$\Rightarrow a\cos\varepsilon = 2.4$$

$$x = a\sin(\omega t + \varepsilon)$$
$$1 = a\sin\varepsilon$$

$$\frac{a\sin\varepsilon}{a\cos\varepsilon} = \frac{1}{2.4}$$

$$\tan\varepsilon = \frac{5}{12} \Rightarrow \varepsilon = 0.395 \text{ rad}$$

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$$a = \frac{1}{\sin 0.395} = 2.6 \text{ m}$$

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