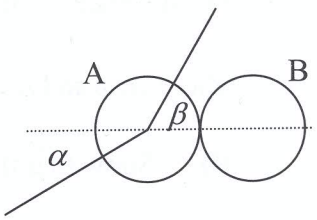
2011

5 (b) A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest.
The direction of motion of A before and after impact makes angles α and β respectively with the line of centres at the instant of impact.



The coefficient of restitution between the spheres is e.

(i) If $\tan\alpha = k\tan\beta$, find k, in terms of e.

(ii) If the magnitude of the impulse imparted to each sphere due to the collision is $\frac{7}{8}mu\cos\alpha$, find the value of e.

(i) PCM $\quad m(u\cos\alpha) + m(0) = mv_1 + mv_2$ — 5

NEL $\quad v_1 - v_2 = -e(u\cos\alpha - 0)$ — 5

$$v_1 = \frac{u\cos\alpha(1-e)}{2}$$

$$v_2 = \frac{u\cos\alpha(1+e)}{2}$$ — 5

$$\tan\beta = \frac{u\sin\alpha}{v_1}$$

$$= \frac{2u\sin\alpha}{u\cos\alpha(1-e)}$$

$$= \frac{2\tan\alpha}{1-e}$$

$$\tan\beta = \frac{2k\tan\beta}{1-e}$$

$$1-e = 2k$$

$$\Rightarrow k = \frac{1-e}{2}$$ — 5

(ii) $\quad I = mv_2 - m(0)$

$$\frac{7}{8}mu\cos\alpha = \frac{1}{2}mu\cos\alpha(1+e)$$

$$e = \frac{3}{4}$$ — 5

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