2011

1. (b) A car accelerates uniformly from rest to a speed v in t_1 seconds. It continues at this constant speed for t seconds and then decelerates uniformly to rest in t_2 seconds.

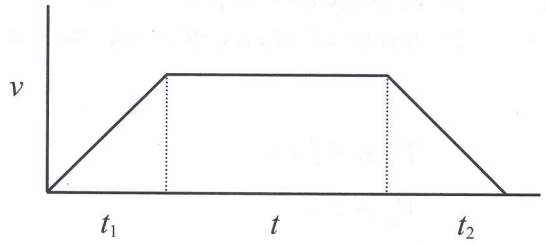
The average speed for the journey is $\frac{3v}{4}$.

(i) Draw a speed-time graph for the motion of the car.

(ii) Find $t_1 + t_2$ in terms of t.

(iii) If a speed limit of $\frac{2v}{3}$ were to be applied, find in terms of t the least time the journey would have taken, assuming the same acceleration and deceleration as in part (ii).

(i)



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(ii) $\text{average speed} = \dfrac{\frac{1}{2}t_1 v + tv + \frac{1}{2}t_2 v}{t_1 + t + t_2}$

$$\frac{3v}{4} = \frac{\frac{1}{2}t_1 v + tv + \frac{1}{2}t_2 v}{t_1 + t + t_2}$$

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$$\frac{3}{4} = \frac{\frac{1}{2}t_1 + t + \frac{1}{2}t_2}{t_1 + t + t_2}$$

$$3t_1 + 3t + 3t_2 = 2t_1 + 4t + 2t_2$$

$$\Rightarrow t_1 + t_2 = t$$

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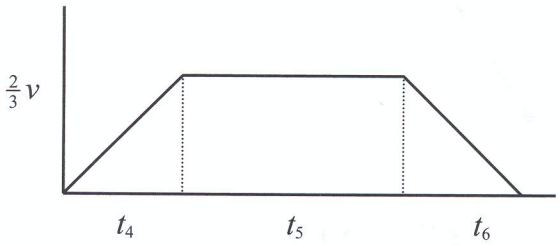

(iii) $\frac{1}{2}t_1 v + tv + \frac{1}{2}t_2 v = \frac{1}{2}t_4\left(\frac{2v}{3}\right) + t_5\left(\frac{2v}{3}\right) + \frac{1}{2}t_6\left(\frac{2v}{3}\right)$

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$$3t_1 v + 6tv + 3t_2 v = 2t_4 v + 4t_5 v + 2t_6 v$$

$$3t_1 + 6t + 3t_2 = 2t_4 + 4t_5 + 2t_6$$

$$9t = 2(t_4 + t_6) + 4t_5$$

$$t_4 + t_6 = \tfrac{2}{3}t$$

$$9t = 2\left(\tfrac{2}{3}t\right) + 4t_5$$

$$\Rightarrow t_5 = \tfrac{23}{12}t$$

$$\Rightarrow t_4 + t_5 + t_6 = \tfrac{2}{3}t + \tfrac{23}{12}t = \tfrac{31}{12}t$$

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