2011

10 (b) A particle travelling in a straight line has a deceleration of

$$\frac{v^2}{400}+16 \quad \text{m s}^{-2}$$

where v is its speed at any time t.

If its initial speed is 40 m s^{-1}, find

(i) the distance travelled before it comes to rest
(ii) the average speed of the particle during the motion.

(i)

$$v\frac{dv}{dx} = -\left(\frac{v^2}{400}+16\right)$$

$$= -\left(\frac{v^2+80^2}{400}\right)$$

$$\int_{40}^{0} \frac{v}{v^2+80^2}\,dv = -\frac{1}{400}\int_0^x dx \qquad 5$$

$$\left[\tfrac{1}{2}\ln\left(v^2+80^2\right)\right]_{40}^{0} = \left[-\frac{x}{400}\right]_0^x \qquad 5$$

$$\tfrac{1}{2}\ln\left(\frac{40^2+80^2}{80^2}\right) = \frac{x}{400}$$

$$x = 200\ln\left(\frac{5}{4}\right)$$

$$x = 44.63 \text{ m} \qquad 5$$

(ii)

$$\frac{dv}{dt} = -\left(\frac{v^2+80^2}{400}\right)$$

$$\int_{40}^{0} \frac{1}{v^2+80^2}\,dv = -\frac{1}{400}\int_0^t dt$$

$$\left[\tfrac{1}{80}\tan^{-1}\left(\frac{v}{80}\right)\right]_{40}^{0} = \left[-\frac{t}{400}\right]_0^t \qquad 5$$

$$t = 5\tan^{-1}\left(\frac{1}{2}\right)$$

$$= 2.32 \qquad 5$$

$$\text{average speed} = \frac{44.63}{2.32} = 19.24 \text{ m s}^{-1} \qquad 5$$

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